MATH 220 Exam 1 **Answers**

1. Assume A is an n x n matrix and $\det(A) = 3$. *(10 pts)*

(a) T **F** $\det(AA) = 1$ $\det(AA) = \det(A)\det(A) = 3 \cdot 3 = 9$

(b) **T** F A has an inverse. $\Leftrightarrow \det(A) \neq 0$

(c) T **F** $\det(A^T) = 1/3$ $\det(A^T) = \det(A) = 3$

(d) T **F** There are three solutions to $Ax = \mathbf{0}$. Here, $\mathbf{0}$ is the column vector of n zeros. There is only one solution $x = A^{-1}0 = 0$

(e) **T** F $\det(A^{-1}) = 1/3 = \dfrac{1}{\det(A)}$

2. Only one answer is correct, circle it. Circle *don't know* and you lose only 3 out of 5 points. *(10 pts)*

- If $B = \begin{pmatrix} -3 & -3 & 4 \\ 0 & 2 & -8 \\ 0 & 0 & 1/2 \end{pmatrix}$, then $\det(B) =$ $(-3)(2)(1/2)$

(a) -3 (b) 2 (c) 0 (d) -6 (e) -1/2 (f) *don't know*

- If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ and $\det(AB) = 1$ then $\det(B) =$ Know $\det(AB) = \underset{\substack{\parallel \\ 1(-2)(2)}}{\det(A)}\det(B)$

$1 = -4 \quad X$

(a) -4 (b) 4 (c) 1/4 (d) -1/4 (e) 0 (f) *don't know*

3. Suppose that $A = \begin{pmatrix} 2 & 3 & 4 \\ -1 & 0 & -1 \\ 4 & 8 & 12 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, and $b = \begin{pmatrix} 0 \\ 1 \\ -6 \end{pmatrix}$. *(5 pts)*

Express the matrix equation $Ax = b$ as a system of equations.

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 0 \\ -x_1 \qquad - x_3 &= 1 \\ 4x_1 + 8x_2 + 12x_3 &= -6 \end{aligned}$$

4. Given that $A = \begin{pmatrix} 1 & -3 & 1 \\ 3 & -8 & 2 \\ 4 & -10 & 2 \end{pmatrix}$. Reduce the matrix to row echelon form, and then to reduced row echelon form and put your answers in the boxes. *(15 pts)*

$\begin{pmatrix} 1 & -3 & 1 \\ 3 & -8 & 2 \\ 4 & -10 & 2 \end{pmatrix}$ $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 4R_1$

$\begin{pmatrix} 1 & -3 & 1 \\ 0 & 1 & -1 \\ 0 & 2 & -2 \end{pmatrix}$ $R_3 \to R_3 - 2R_1$

$\begin{pmatrix} 1 & -3 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ ✱ - row echelon form

$R_1 \to R_1 + 3R_2$

$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$ ✱ - reduced row echelon form

5. Find the determinant of the matrix A. *(10 pts)*

Reminder: Justify your answer. Use a sentence if you have no calculations to demonstrate.

$$A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -4 \end{pmatrix} \underset{\text{switch rows}}{=} -1 \begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -4 \end{vmatrix} = -1(2)(-1)(3)(-4) = -24$$

$\boxed{\det(A) = -24}$

6. Find the inverse of the matrix A. *(10 pts)*

$$A = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -4 \end{pmatrix}$$

$$\left[\begin{array}{cccc|cccc} 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -4 & 0 & 0 & 0 & 1 \end{array}\right] \quad R_2 \leftrightarrows R_3$$

$$\left[\begin{array}{cccc|cccc} 2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_1 \to \frac{1}{2}R_1 \\ R_2 \to -1R_2 \\ R_3 \to 1/3\, R_3 \\ R_4 \to -\frac{1}{4}R_4 \end{array}$$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1/4 \end{array}\right]$$

$\qquad I \qquad\qquad A^{-1}$

$$\boxed{A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & -1/4 \end{bmatrix}}$$

7. For what values of α is the matrix $\begin{pmatrix} \alpha - 1 & 2 \\ 1 & \alpha - 2 \end{pmatrix}$ not invertible? *(10 pts)*

not invertible $\Leftrightarrow$ $\det(A) = 0$

$$\begin{aligned} \det(A) &= (\alpha-1)(\alpha-2) - 2 \\ &= \alpha^2 - 3\alpha + 2 - 2 \\ &= \alpha^2 - 3\alpha \\ &= \alpha(\alpha-3) \end{aligned}$$

This equals zero when $\alpha = 0$ or $\alpha = 3$

your answers:

$\alpha = 0$ or $\alpha = 3$

8. Assume A and B are two 2 x 2 matrices. Show that, in general, it is not true that $AB = BA$. *(10 pts)*

Almost any two matrices will work

$$\underset{A}{\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}} \underset{B}{\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}} = \underset{AB}{\begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix}}$$

So for $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ + $B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$

$AB \neq BA$

$$\underset{B}{\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}} = \underset{BA}{\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}}$$