MATH 220 Exam 1 **Answers**

1. Assume A is an n x n matrix and $\det(A) = 4$. *(10 pts)*

   (a) **T** **F** $\det(AA) = 4$ $\det(AA) = \det(A)\cdot\det(A) = 4\cdot 4 = 16$

   (b) **T** **F** A does <u>not</u> have an inverse. $\det(A) \neq 0 \Leftrightarrow A$ has an inverse

   (c) **T** **F** $\det(A^T) = 4$ yes $\det(A^T) = \det(A)$

   (d) **T** **F** $x = \mathbf{0}$ (the zero vector) is the only solution to $Ax = \mathbf{0}$.

   (e) **T** **F** $\det(A^{-1}) = 4$ $\det(A^{-1}) = \frac{1}{\det(A)} = \frac{1}{4}$

2. Let $B = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$. Find the determinant of B. *(5 pts)*

   switch $r_1 = -1$

   Switch rows 1 & 2 get a multiplier of $r_1 = -1$. The determinant of $U = \begin{pmatrix} 2&0&0&0\\0&1&0&0\\0&0&4&0\\0&0&0&3\end{pmatrix} = 2\cdot1\cdot4\cdot3 = 24$ so $\det(U) = r_1 \det(B)$

   $24 = -\det(B)$

   $\boxed{\det B = -24}$

3. For what values of λ is the matrix $\begin{pmatrix} 1-\lambda & -2 \\ 1 & 4-\lambda \end{pmatrix}$ not invertible? set determinant $= 0$ *(5 pts)*

   $\det(A) = (1-\lambda)(4-\lambda) + 2$
   $= \lambda^2 - 5\lambda + 4 + 2$
   $\rightarrow \det(A) = \lambda^2 - 5\lambda + 6$
   $= (\lambda - 3)(\lambda - 2) = 0$

   $\lambda = 3$
   $\lambda = 2$

4. Consider the problem of finding the coefficients in the quadratic function $p(x) = ax^2 + bx + c$ if the graph of the resulting parabola passes through the points $(1, 5)$, $(0, 2)$, and $(2, 10)$. *(20 pts)*

   (a) Solve this problem by setting it up as a system of 3 equations in 3 unknowns. Write the augmented matrix which results from this process.

   (b) Using the augmented matrix from part (a), determine the coefficients a, b, and c.

$x = 1,\ p(x) = 5 \rightarrow a + b + c = 5$

$x = 0,\ p(x) = 2 \rightarrow c = 2$

$x = 2,\ p(x) = 10 \rightarrow 4a + 2b + c = 10$

$\rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 4 & 2 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 10 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 0 & 1 & 2 \\ 4 & 2 & 1 & 10 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 4 & 2 & 1 & 10 \\ 0 & 0 & 1 & 2 \end{array}\right]$ $R_2 \rightarrow R_2 - 4R_1$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & -2 & -3 & -10 \\ 0 & 0 & 1 & 2 \end{array}\right]$ $R_2 \rightarrow -\frac{1}{2}R_2$

$\rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3/2 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right] \rightarrow a + b + c = 5$

$a + 2 + 2 = 5$

$\boxed{a = 1}$

$\boxed{c = 2}$

$b + \frac{3}{2}c = 5$

$b + 3 = 5$

$\boxed{b = 2}$

$\begin{bmatrix} a = 1 \\ b = 2 \\ c = 2 \end{bmatrix}$