

Test 1 Math 152 Section 6

Name:

Each answer after #1 is worth five points.

1. Consider the function $f(x) = \frac{x^2 - 4}{x - 2}$, which has a *removable discontinuity* at $x = 2$.
 - (a) Show work that finds the exact value for $\lim_{x \rightarrow 2} f(x)$.

 - (b) What value should be assigned to $f(2)$ to remove the discontinuity?

2. Use the fact that $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$ to find $\lim_{t \rightarrow 0} \frac{\sin(3t)}{t}$.

3. Use the **Intermediate Value Theorem** to answer the following questions
 - (a) Prove the equation $x^3 - 4x + 1 = 0$ has at least one solution in the interval $[1,2]$.

 - (b) Prove there is a number that is exactly one more than it's cube (x^3).
Hint: Express the above statement as an equation.

4. Use the *Squeezing Theorem* to show that $\lim_{x \rightarrow 0} x^2 \cos\left(\frac{50\pi}{x}\right) = 0$ and sketch the graph illustrating this.

5. State the formal (limit) definition of the derivative of a function f with respect to x .

6. I know, and hopefully you know, that if $f(x) = x^2 - x$ then $f'(x) = 2x - 1$. Prove to me that this is true by evaluating the formal (limit) definition of the derivative.
7. Suppose f is differentiable over all real numbers. Also suppose $f(4) = 5$ and $f'(4) = -3$. Write an equation for the line that is tangent to the graph of f at the point $(4, f(4))$.
8. A rock is dropped from a height of 576 ft and falls toward Earth in a straight line. In t seconds the rock drops a distance of $s = 16t^2$.
- (a) Show that the rock hits the ground 6 seconds after release.
 - (b) What is the average velocity of the rock during the time it is falling?
 - (c) What is the instantaneous velocity of the rock when it hits the ground?
9. In the text they state that under appropriate conditions on g ,

$$\frac{d}{dx} \left[\frac{1}{g(x)} \right] = \frac{-\frac{d}{dx} [g(x)]}{[g(x)]^2}.$$

Prove this result using the quotient rule.

10. Find the derivative of the following functions and simplify your answer.

(a) $g(x) = (4x^3 - x^2 + 2)^8$

(b) $f(t) = \sin^3(t)$

(c) $g(t) = \sin^3(5t^2)$

(d) $f(x) = x^2 \cos(x)$