Test 2 Math 152 Section 6 Name: ....
Show relevant work for full credit.

1. Suppose a continuous function $f(x)$ defined over all real numbers is increasing on $(-\infty, 0]$ and decreasing on $[0, \infty)$. Does this function have an inverse? Why or why not? *5 pts*

2. Consider the function $f(x) = \sqrt[2]{x-2}$. Find the domain and range of $f^{-1}(x)$ based on the domain and range of $f(x)$.

   5 pts

3. Solve for x without using a calculating utility. *10 pts*

   (a) $e^x - 2xe^x = 0$ (b) $\log_3 3^x = 7$

4. Consider the circle defined by the equation $x^2 + y^2 = 4$. *10 pts*

   (a) Find dy/dx by implicit differentiation.

   (b) Show that the tangent line at any point (x_o, y_o) on the circle is perpendicular to the line from the origin to that point.

5. Find the derivatives of the following functions. *10 pts*

(a) $f(t) = \cos(\ln t)$, $\quad t > 0$ (b) $f(x) = e^{(x - e^{(3x)})}$

6. Find a formula for the n'th derivative, $f^{(n)}(x)$, of the function $f(x) = e^{bx}$.

5 pts

7. Given that $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$, find $\frac{d}{dx}[\tan^{-1}(e^{2x})]$ and simplify your answer.

5 pts

8. When I pour baby oil into Nicolas' tub the slick spreads out in a circular region that is 2 mm deep. If I am pouring oil into the tub at a constant rate of 400 mm^3/sec, how fast is the radius of the slick increasing when the radius of the slick is 50 mm. Give the dimensions of your answer.

5 pts

9. A 13-ft ladder is leaning against a wall. If the top of the ladder slips down the wall at a rate of 2 ft/sec, how fast will the foot be moving away from the wall when the top is 5 ft above the ground. Give the dimensions of your answer.

5 pts

10. Use L'Hopital's rule to find the following limits. *6 pts*

(a) $\lim_{x\to\infty} \frac{\ln x}{x}$ (b) $\lim_{x\to 0} \frac{e^x - 1}{\sin x}$

11. Find the following limit by first taking the natural logarithm of the expression. *4 pts*

$\lim_{x\to\infty} (\ln x)^{1/x}$

12. The graph of a function $f(x)$ is depicted below, with inflection points at $(-1, 4)$ and $(1, 4)$. *15 pts*

(a) On what intervals is $f(x)$ increasing?

(b) On what intervals is $f(x)$ decreasing?

(c) On what intervals is $f(x)$ concave up?

(d) On what intervals is $f(x)$ concave down?

(e) What is the relative maximum of $f(x)$?

(f) Sketch the graph of $f'(x)$ below the graph of $f(x)$.

13. Consider the function $f(x) = x^3 - 3x + 3$. *10 pts*

(a) Use the first derivative test to show there is relative minimum at $x = 1$ and a relative maximum at $x = -1$.

(b) Use the second derivative test to check the results in part (a).

14. Consider the function $f(x) = \frac{1}{2}x^2 + 3x^{1/3}$. *5 pts*

(a) What are the x values for the critical points of $f(x)$?

(b) Label these as to whether the resulting point is stationary or a point of nondifferentiability.