

Show relevant work for full credit.

1. Consider the function $f(x) = \frac{1}{x(1-x)}$. *10 pts*

(a) Convince me that this function has no absolute maximum on the interval $(0,1)$.

(b) What is the absolute minimum of this function and where does it occur. Justify your answer.

2. A container with square base, vertical sides, and open top is to be made from 2700 ft² of material. Find the dimensions of the container with greatest volume. *10 pts*

3. A man is on the bank of a river that is 1 mile wide. He wants to travel to a town on the opposite bank, but 1 mile upstream. He intends to row on a straight line to some point P on the opposite bank and then walk the remaining distance along the bank (see figure). If he can walk 5 mi/hr and row 3 mi/hr, to what point should he row in order to reach his destination in the least time. **Hint** Use the formula (distance = rate x time), to solve for the total time required in terms x . *10 pts*

4. The accompanying figure shows a graph of distance (s) versus time (t) of a particle moving along a coordinate line. *10 pts*

(a) When, if ever, is the particle to the right of its starting position?

(b) When, if ever, does the particle have zero velocity?

(c) Sketch the velocity curve on the same graph.

5. Use Rolle's theorem to prove that if f is differentiable over all real numbers and the graph of f intersects the x-axis at the points $x = a$ and $x = b$, then the graph of f' has at least one x-intercept between a and b . *5 pts*

6. Let $f(x) = \tan x$. *5 pts*

(a) Show that there is no point c in the interval $(0, \pi)$ such that $f'(c) = 0$, even though $f(0) = 0$ and $f(\pi) = 0$.

(b) Explain why the result in part (a) does not violate the mean-value theorem.

7. Consider the function $f(x) = x^2$ graphed over the interval $[0,4]$.

10 pts

- (a) Use the **right endpoint method** to draw 4 rectangles of equal base such that the sum the area of the 4 rectangles approximates the area under the curve.
- (b) Will the estimate with this technique be too large or too small?
- (c) What is the estimated area using these rectangles?
- (d) Use the fact that $\frac{d}{dx} \left(\frac{x^3}{3} \right) = x^2$ to find the exact area under the curve.

8. Evaluate the following indefinite integrals.

20 pts

(a) $\int x(x + x^3) dx$

(b) $\int \left(\frac{2}{x} + e^{2x} \right) dx$

(c) $\int t\sqrt{7t^2 + 12} dt$

(d) $\int \sin^3 x \cos x dx$

9. Solve the initial value problem: $\frac{dy}{dx} = \sqrt[3]{x}$, $y(1) = 2$ 5 pts

10. Find a function f such that the slope of the tangent line at a point (x, y) on the curve $y = f(x)$ is $\sqrt{3x+1}$, and the curve passes through the point $(0,1)$. 5 pts

11. Find $\int_1^5 f(x) dx$ if $\int_0^1 f(x) dx = -2$ and $\int_0^5 f(x) dx = 1$. 4 pts

12. Evaluate the limit over the interval $[-2,2]$ by expressing it as a definite integral and applying an appropriate formula from geometry. 6 pts

$$\lim_{\max \Delta x_k \rightarrow 0} \sum_{k=1}^n \sqrt{4 - (x_k^*)^2} \Delta x_k$$