MA153, Section 1, TEST 1 **Answers**

1. Find a function f such that the slope of the tangent line at a point (x, y) on the curve $y = f(x)$ is $\sqrt{3x+1}$ and the curve passes through the point $(0, 1)$. *(10 pts)*

2. Suppose that sludge is emptied into a river at a rate of $V(t)$ gallons per minute, starting at time $t = 0$. Write an integral equation that represents the total volume of sludge that is emptied into the river during the first hour. *(5 pts)*

3. Express the following limit as a definite integral over the interval [0,3]. Do not try to evaluate the integral. *(5 pts)*

$$\lim_{max\ \Delta x_k \to 0} \sum_{k=1}^{n} \sqrt{4 - (x_k^*)^2}\ \Delta x_k =$$

4. In this problem you will setup an approximation of the definite integral, $\int_0^2 x^2\, dx$, using the **right endpoint** approximation with **four** equal-width subintervals. Note, the "integrand" is the function $f(x) = x^2$. *(25 pts)*

   (a) Graph the integrand over the interval [0,2].

   (b) What is the value of Δx in this problem?

   (c) List the right endpoints of each interval.

   (d) Evaluate the integrand at each of the end points and list the results.

   (e) Write a Riemann sum describing the approximation. You need not use summation notation.

   (f) Will the above Riemann sum yield an approximation that is too small or too large. Justify your answer by displaying the geometric interperetation of the sum from part (e) on the graph from part (a).

   (g) What is the limiting value of the approximation as the number of equal-width intervals tends to infinity. Exact answer, not a decimal approximation.

   (h) Write out the theorem that allows you to make the conclusion in part (g).

5. Consider the region enclosed between $y = x(1-x)$ and the x-axis. *(20 pts)*

   (a) Sketch the enclosed region.

   (b) What is the best method for determining the volume of the solid generated by rotating the region about the y-axis? Why?

   (c) Set up but do not evaluate the integral describing the volume in part (b).

   (d) Set up but do not evaluate the integral describing the volume generated by revolving the region around the x-axis.

6. A nose cone for a space reentry vehicle is designed so that a cross section, taken x ft from the tip and perpendicular to the axis of symmetry, has an area $A(x) = \pi x \ \text{ft}^2$. Find the volume of the nose cone given that it's length is 20 ft. *(10 pts)*

7. Show that the circumference of a circle with radius r is $2\pi r$ by calculating the arclength of the parametric equations:

   $x(t) = r\cos t, \qquad y(t) = r\sin t, \qquad 0 \leq t \leq 2\pi$ *(10 pts)*

8. Write an integral expression for the area of the region enclosed by $y = x + 2$ and $y = x^2$. *(10 pts)*

9. Find the volume of the solid that is generated when the region enclosed by $y = \cosh 2x$, $y = \sinh 2x$, $x = 0$ and $x = 3$ is revolved about the x-axis. *(10 pts)*