MA153, Section 1, Part 2, TEST 2 (60 pts) **Answers**

1. Here we investigate the improper integral $\int_0^1 ln\, x \;\, dx$. *(16 pts)*

   (a) What makes the above definite integral improper?

   (b) Use integration by parts to show that $\int lnx \; dx = x\ln(x) - x + C$.

   (c) Use part (b) to express the improper integral $\int_0^1 ln\, x \;\, dx$ as a limit.

   (d) Does the improper integral converge or diverge. If it converges, to what does it converge. Hint: you will need to apply L'Hopital's rule to the expression $\lim_{x\to 0^+} x\ln(x) = \lim_{x\to 0^+} \frac{\ln x}{1/x}$.

2. (a) Confirm graphically or algebraically that $\frac{1}{x^2+3x+2} \le \frac{1}{x^2}$ for $x \ge 1$. *(4 pts)*

   (b) Evaluate the integral the improper integral $\int_1^\infty \frac{1}{x^2}\, dx$. *(4 pts)*

   (c) Use parts (a) and (b) to conclude whether $\int_1^\infty \frac{1}{x^2+3x+2}\, dx$ converges or diverges. *(4 pts)*

3. Express the following statements as a differential equation. Be sure to state what each variable stands for and whether any constants are positive or negative. *(8 pts)*

   (a) *The rate at which the temperature of a warming turkey increases in an oven is proportional to the difference between the oven temperature and the temperature of the turkey.*

   (b) *The amount of a drug that is present in the blood stream tends to decrease at a rate that is proportional to the amount present.*

4. At time $t = 0$, a tank contains 25 ounces of salt dissolved in 50 gal of water. Then brine containing 4 ounces of salt per gallon of brine is allowed to enter the tank at a rate of 2 gal/min and the mixed solution is drained from the tank at the same rate. Let $y(t)$ denote the amount of salt in the tank at time $= t$. *(12 pts)*

(a) Set up the initial value problem describing the rate of change in y with respect to time.

(b) Solve the initial value problem for $y(t)$.

(c) As $t \to \infty$ what does the **concentration** of salt in the tank tend towards.

5. A colony of the bacterium E. *coli* grows at a relative rate of 80 percent per hour when placed in a nutrient culture. Let $y = y(t)$ be the number of cells that are present after 100 cells are placed in the culture. *(12 pts)*

(a) Find an initial value problem whose solution is $y(t)$.

(b) Find a formula for $y(t)$, where all constants are assigned a numerical value.

(c) How long does it take for the number of cells to double.