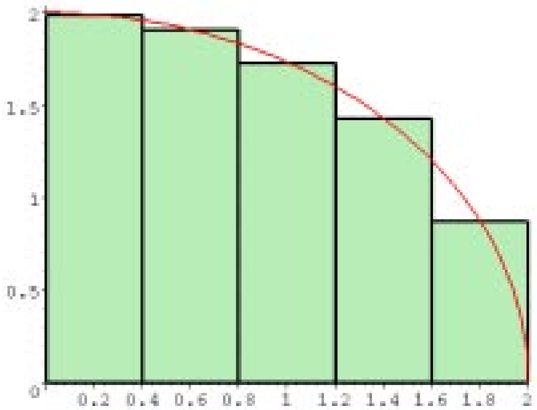
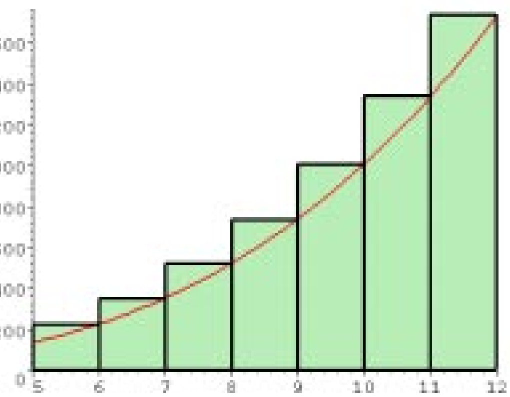
1. Partition the interval $[0,2]$ into n subintervals. Let Δx_k be the width of the k^{th} subinterval and let x_k^* be some point in that subinterval, then $\sum_{k=1}^{n}\left(\sqrt{4-(x_k^*)^2}\cdot\Delta x_k\right)$ is a Riemann sum that approximates $\int_0^2 \sqrt{4-x^2}\,dx$; furthermore the value of that definite integral is the limit of such Riemann sums. The region above the x-axis and below the graph of $y=\sqrt{4-x^2}$ over the interval $[0,2]$ is one-quarter of a circle with radius 2, hence its area and the value of the definite integral equals $(1/4)\cdot\pi\cdot 2^2=\pi$. (The left-hand plot below shows this region, $n=5$ equal-width subintervals, and rectangles with heights computed at midpoints of subintervals.)





2. Partition the interval $[5,12]$ into n equal-width subintervals, then $\Delta x=(12-5)/n=7/n$. The first (left-hand) subinterval is $[5,5+\Delta x]$, the second subinterval is $[5+\Delta x,5+2\cdot\Delta x]$; in general, the right endpoint of the k^{th} subinterval is $x_k=5+k\cdot\Delta x$. The sum contains the expression $7+(5+k\cdot\Delta x)^3$, that is just $7+(x_k)^3$. The sum $\sum_{k=1}^{n}\left(7+(5+k\cdot\Delta x)^3\right)\cdot\Delta x=\sum_{k=1}^{n}\left(7+x_k^3\right)\cdot\Delta x$ is a Riemann sum that approximates $\int_5^{12}\left(7+x^3\right)\,dx$.
The right-hand plot above shows the region between the x-axis and the graph of $y=7+x^3$ over the interval $[5,12]$ together with some rectangles whose heights are computed at the right endpoint of their base subinterval.

Note: Theorem 7.4.2 (page 400) implies $\sum_{k=1}^{n}\left(7+\left(5+k\cdot\frac{7}{n}\right)^3\right)\cdot\frac{7}{n}=\left(\frac{7}{4}\right)\cdot\left(\frac{2901\,n^2+3206\,n+833}{n^2}\right)$.

3. Partition the interval $[0,6]$ into a collection of subintervals. For each subinterval, evaluate f at some point of this subinterval, multiply that function value by the width of this subinterval; finish by adding all of those products. We have a table of values for $f(x)$ at integer values of x. It is convenient to use subintervals whose endpoints are integers; there are many ways to do that. Here are several alternative approximations of $\int_0^6 f(x)\,dx$.

$\Delta x=3$, left endpoint	$f(0)\cdot 3+f(3)\cdot 3=(8.8)\cdot 3+(2.8)\cdot 3=26.4+8.4=34.0$
haphazard	$f(2)\cdot 3+f(4)\cdot 2+f(6)\cdot 1=(2.4)\cdot 3+(4.0)\cdot 2+(4.0)\cdot 1=19.2$
$\Delta x=2$, midpoint	$f(1)\cdot 2+f(3)\cdot 2+f(5)\cdot 2=(4.0)\cdot 2+(2.8)\cdot 2+(4.8)\cdot 2=8.0+5.6+9.6=23.2$
$\Delta x=1$, left endpoint	$f(0)+f(1)+f(2)+f(3)+f(4)+f(5)=8.8+4.0+2.4+2.8+4.0+4.8=26.8$
$\Delta x=1$, right endpoint	$f(1)+f(2)+f(3)+f(4)+f(5)+f(6)=4.0+2.4+2.8+4.0+4.8+4.0=22.0$

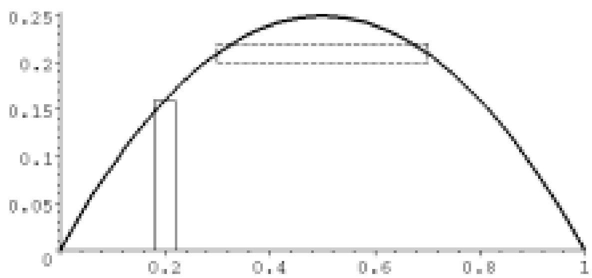
4. a) $x\cdot(1-x)$ equals 0 at the ends of the interval $[0,1]$, is positive in the interior of that interval, and is negative outside. The left-hand plot below shows the graph of $y=x\cdot(1-x)$ over $[0,1]$.

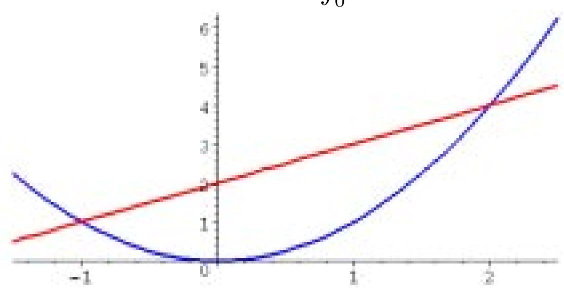
   b) Whether we rotate the region about the y-axis or the x-axis, it is moderately simple to begin by setting up a Riemann sum that involve computing what happens to a thin vertical rectangle that goes from 0 to the upper bounding curve: the height of such a rectangle is easy to compute as a function of x. On the other hand, beginning with a thin horizontal rectangle seems messier: its left end computed as one function of x and its right end as another function of x.

   c) Suppose x is in $[0,1]$. A thin vertical strip at x, thickness Δx and height $x\cdot(1-x)$, is distance x from the y-axis; rotating this strip around the y-axis will generate a cylindrical shell with (approximate) volume

$2\,\pi \cdot x \cdot \Big(x \cdot (1-x)\Big) \cdot \Delta x$. The total volume of all shells is computed by $\sum_{k=1}^{n} \Big(2\,\pi \cdot x_k \cdot \big(x_k \cdot (1-x_k)\big) \cdot \Delta x_k\Big)$, a Riemann sum whose limit is the definite integral $\int_0^1 2\,\pi\, x \cdot \Big(x \cdot (1-x)\Big)\ dx$.

d) If region rotated is about x-axis, the disk method yields the definite integral $\int_0^1 \pi \cdot \Big(x \cdot (1-x)\Big)^2\ dx$.





5. The right-hand plot above shows graphs of the straight line $y_1 = x+2$ and the parabola $y_2 = x^2$. Those curves meet at the points $(-1,1)$ and $(2,4)$; the straight line lies above the parabola over the interval $[-1,2]$. The area of the region between those curves is computed by the definite integral $\int_{-1}^{2} \Big((x+2) - x^2\Big)\ dx$

6. a) $\displaystyle\int \frac{1}{(x-1)\cdot(x+1)}\ dx = \int \left(\frac{1/2}{x-1} - \frac{1/2}{x+1}\right)\ dx = \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C.$

b) $\displaystyle\int \frac{1}{(x-1)\cdot(x^2+1)}\ dx = \frac{1}{2}\int \left(\frac{1}{x-1} - \frac{1}{x^2+1} - \frac{x}{x^2+1}\right)\ dx = \frac{1}{2}\ln|x-1| - \frac{1}{2}\arctan(x) - \frac{1}{4}\ln\left(x^2+1\right) + C.$

7. The substitution $x = \tan(\theta)$ implies $dx = \sec(\theta)^2\ d\theta$ and

$$\int \sqrt{1+x^2}\ dx = \int \sqrt{1+\tan(\theta)^2} \cdot \Big(\sec(\theta)^2\ d\theta\Big) = \int \sqrt{\sec(\theta)^2} \cdot \sec(\theta)^2\ d\theta = \int |\sec(\theta)| \cdot \sec(\theta)^2\ d\theta = \int \sec(\theta)^3\ d\theta.$$

Intermediate steps in the preceding equation involve the identities $1+\tan(\theta)^2 = \sec(\theta)^2$ and $\sqrt{w^2} = |w|$; the final step is a consequence of $\theta = \arctan(x)$ and $\sec(\theta) > 0$ on $(-\pi/2, \pi/2)$, the range of the arctangent function.

Now use integration by parts: $\int A \cdot dB = A \cdot B - \int B \cdot dA$. Let $A = \sec(\theta)$ and $dB = \sec(\theta)^2\ d\theta$, then $dA = \sec(\theta) \cdot \tan(\theta)\ d\theta$, $B = \tan(\theta)$ and

$$\begin{aligned}
\int \sec(\theta)^3\ d\theta &= \int \sec(\theta)\ \cdot\ d\Big(\tan(\theta)\Big) \\
&= \sec(\theta) \cdot \tan(\theta) - \int \tan(\theta) \cdot \Big(\sec(\theta) \cdot \tan(\theta)\ d\theta\Big) \\
&= \sec(\theta)\tan(\theta) - \int \sec(\theta) \cdot \tan(\theta)^2\ d\theta \\
&= \sec(\theta)\tan(\theta) - \int \sec(\theta) \cdot \left(-1 + \sec(\theta)^2\right)\ d\theta \\
&= \sec(\theta)\tan(\theta) + \int \sec(\theta)\ d\theta - \int \sec(\theta)^3\ d\theta.
\end{aligned}$$

Solve the preceding equation for $\int \sec(\theta)^3\ d\theta$ and evaluate the remaining integral:

$$\int \sec(\theta)^3\ d\theta = \frac{1}{2} \cdot \left(\sec(\theta)\tan(\theta) + \int \sec(\theta)\ d\theta\right) = \frac{1}{2} \cdot \Big(\sec(\theta)\tan(\theta) + \ln\big|\sec(\theta)+\tan(\theta)\big|\Big) + C.$$

Now we are ready to calculate a numerical value. The integral with respect to x over the interval $[0,1]$ is transformed into an integral with respect to θ over the interval $[\arctan(0), \arctan(1)] = [0, \pi/4]$. Note that $\sec(0) = 1$, $\tan(0) = 0$, $\sec(\pi/4) = \sqrt{2}$, $\tan(\pi/4) = 1$ and the value of the definite integral is

$$\frac{1}{2} \cdot \Big(\sqrt{2} \cdot 1 + \ln\big(\sqrt{2}+1\big)\Big) - \frac{1}{2} \cdot \big(1 \cdot 0 + \ln(1+0)\big) = \frac{1}{\sqrt{2}} + \frac{1}{2}\ln\left(\sqrt{2}+1\right).$$

Section 8.8 provides an alternative substitution for problem 7: $x = \sinh(t)$ implies $dx = \cosh(t)\,dt$, $\sqrt{1+x^2} = \sqrt{1+\sinh(t)^2} = \sqrt{\cosh(t)^2} = \big|\cosh(t)\big| = \cosh(t)$, and

$$\int \sqrt{1+x^2}\;dx = \int \cosh(t)\cdot\Big(\cosh(t)\;dt\Big) = \int \cosh(t)^2\;dt = \frac{1}{2}\cdot\Big(t+\sinh(t)\cdot\cosh(t)\Big)+C.$$

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8. If $y = (1/2)\,x^2$, then $y\,' = x$ and $\sqrt{1+(y\,')^2} = \sqrt{1+x^2}$; the definite integral $\displaystyle\int_0^2 \sqrt{1+x^2}\;dx$ computes the arclength over the interval $[0,2]$. Calculate the numerical value by adapting the solution to the preceding problem. The definite integral with respect to θ is over the interval $\big[\arctan(0),\arctan(2)\big] = \big[0,\arctan(2)\big]$. The identity $\sec(\theta)^2 = \tan(\theta)^2+1$ implies $\sec(\arctan(2)) = \sqrt{5}$. The arclength is $\displaystyle\frac{1}{2}\cdot\Big(\sqrt{5}\cdot 2+\ln\Big(\sqrt{5}+2\Big)\Big)$.

9. a) $\displaystyle\int_0^1 \ln(x)\;dx$ is an improper integral because 0 is not in the domain of the natural logarithm function and $\displaystyle\lim_{x\to 0^+}\ln(x) = -\infty$.

   b) Let $A = \ln(x)$ and $dB = dx$, then $dA = (1/x)\,dx$, $B = x$, and
   $$\int \ln(x)\;dx = \ln(x)\cdot x - \int x\cdot d\Big(\ln(x)\Big) = \ln(x)\cdot x - \int x\cdot\left(\frac{1}{x}\;dx\right) = x\cdot\ln(x) - \int dx = x\cdot\ln(x)-x+C.$$

   c) $$\int_0^1 \ln(x)\;dx = \lim_{L\to 0^+}\int_L^1 \ln(x)\;dx = \lim_{L\to 0^+}\left(x\cdot\ln|x|-x\right)\bigg|_{x=L}^{1} = -1-\lim_{L\to 0^+} L\cdot\ln(L)$$

   d) $\displaystyle -L\cdot\ln(L) = \frac{-\ln(L)}{1/L}$ is indeterminate of type $\displaystyle\frac{\infty}{\infty}$ as $L\to 0^+$. The derivative of the numerator is $-1/L$, the derivative of the denominator is $-1/L^2$ and it is never equal to 0 in $(0,1)$, and $\displaystyle\lim_{L\to 0^+}\frac{-1/L}{-1/L^2} = \lim_{L\to 0^+} L = 0$; hence l'Hôpital's Rule implies $\displaystyle\lim_{L\to 0^+}\Big(-L\cdot\ln(L)\Big) = \lim_{L\to 0^+}\frac{-\ln(L)}{1/L}$ also exists and equals 0. Hence $\displaystyle\int_0^1 \ln(x)\;dx = -1$.

10. Let $S(t)$ be the ounces of salt in the tank at time t.

   a) Salt flows in and salt flows out. The net rate-of-change for $S(t)$ can be decomposed:
   $$\frac{dS}{dt} = \begin{bmatrix}\text{rate}\\ \text{in}\end{bmatrix} - \begin{bmatrix}\text{rate}\\ \text{out}\end{bmatrix} = \left(2\;\frac{\text{gal}}{\text{min}}\right)\cdot\left(4\;\frac{\text{ounces}}{\text{gal}}\right) - \left(2\;\frac{\text{gal}}{\text{min}}\right)\cdot\left(\frac{S(t)\text{ ounces}}{50\text{ gal}}\right) = 8-\frac{1}{25}\cdot S(t).$$
   Since there are 25 ounces of salt in the tank at time $t=0$, we have the initial condition $S(0) = 25$.

   b) The differential equation $S\,'(t) = 8-(1/25)\,S(t)$ can be rewritten as $S\,'(t)+(1/25)\,S(t) = 8$ for which $\mu = e^{\int (1/25)\;dt} = e^{t/25}$ is an integrating factor. Using it, we find
   $$\frac{d}{dt}\Big(e^{t/25}\cdot S(t)\Big) = e^{t/25}\cdot S\,'(t)+\frac{1}{25}\,e^{t/25}\,S(t) = 8\cdot e^{t/25} = \frac{d}{dt}\Big(8\cdot 25\cdot e^{t/25}\Big)$$
   and infer $S(t) = 8\cdot 25+C\cdot e^{-t/25}$ where C is an arbitrary constant.

   The initial condition implies $25 = S(0) = 8\cdot 25+C\cdot e^0 = 200+C$ and $C = 25-200 = -175$. Hence the Initial Value Problem has solution $S(t) = 200-175\cdot e^{-t/25}$ for $t\geq 0$.

   c) $\displaystyle\lim_{t\to\infty} S(t) = 200$ ounces of salt and the limiting concentration of salt is $\displaystyle\lim_{t\to\infty}\frac{S(t)}{50} = 4$ ounces-per-gallon. Note that the limit of the tank's concentration is equal to the concentration in the inflow.

11. a) $\displaystyle\frac{1}{x}$ is a positive and decreasing function on $[1,\infty)$, the improper integral $\displaystyle\int_1^\infty \frac{dx}{x} = \lim_{R\to\infty}\ln(R) = \infty$ diverges, therefore so does $\displaystyle\sum_{k=1}^\infty \frac{1}{k}$.

   b) $\displaystyle\frac{5}{e^{x/2}} = 5\cdot e^{-x/2}$ is a positive and decreasing function on $[0,\infty)$, the improper integral $\displaystyle\int_0^\infty 5\cdot e^{-x/2}\;dx = 10-10\cdot\lim_{R\to\infty} e^{-R/2} = 10$ converges, therefore so does $\displaystyle\sum_{k=1}^\infty \frac{5}{e^{k/2}}$.

   This is a convergent geometric series: $\displaystyle\sum_{k=1}^\infty \frac{5}{e^{k/2}} = \frac{5}{\sqrt{e}}\cdot\sum_{k=0}^\infty\left(\frac{1}{\sqrt{e}}\right)^k = \frac{5}{\sqrt{e}}\cdot\frac{1}{1-1/\sqrt{e}} = \frac{5}{\sqrt{e}-1}$.

12. Begin by citing our main result about a geometric series:

$$\sum_{n=0}^{\infty} r^n \textbf{ converges if and only if } |r|<1; \quad \textbf{if this series converges, then its limit is } \frac{1}{1-r}\textbf{.}$$

Analyze $\displaystyle\sum_{n=0}^{\infty}\left(x-\frac{3}{2}\right)^n$ by letting $r = x - \frac{3}{2}$ and adapting the above statements to get ones involving x.

a) The series converges at $x = 1$ because $|1 - 3/2| = 1/2 < 1$; the series diverges at $x = 3$ because $|3 - 3/2| = 3/2 > 1$.

b) The series converges if and only if $|x - 3/2| < 1$; that inequality is equivalent to $3/2 - 1 < x < 3/2 + 1$. The interval of convergence for this series is $(1/2, 5/2)$.

c) If w is in the interval of convergence, then $\displaystyle\sum_{n=0}^{\infty}\left(w-\frac{3}{2}\right)^n = \frac{1}{1-(w-3/2)} = \frac{2}{5-2\,w}$.

13. Alternating Series Test: If $\{A_k\}_{k=1}^{\infty}$ is a sequence of positive terms which (a) decrease and (b) tend to 0 as $k \to \infty$, then $\displaystyle\sum_{k=1}^{\infty}(-1)^{k-1}\,A_k$ converges.

Suppose $A_k = \dfrac{1}{k}$. If k is positive, then $k+1 > k$ implies $A_k = \dfrac{1}{k} > \dfrac{1}{k+1} = A_{k+1}$. Furthermore, $\displaystyle\lim_{k\to\infty} A_k = \lim_{k\to\infty}\frac{1}{k} = 0$. Therefore $\displaystyle\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ converges.

14. a) $\dfrac{3^k}{k!}$ is positive for each positive integer k and $\displaystyle\lim_{k\to\infty}\frac{3^{k+1}/(k+1)!}{3^k/k!} = \lim_{k\to\infty}\frac{3^{k+1}}{3^k}\cdot\frac{k!}{(k+1)!} = \lim_{k\to\infty}\frac{3}{k+1} = 0$; since that limit is less than 1, the Limit Ratio Test implies $\displaystyle\sum_{k=1}^{\infty}\frac{3^k}{k!}$ converges.

b) $\displaystyle\lim_{k\to\infty}\frac{1}{5+4^{-k}} = \frac{1}{5+0} = \frac{1}{5}$ is not equal to zero, therefore $\displaystyle\sum_{k=1}^{\infty}\frac{1}{5+4^{-k}}$ does not converge.

15. A function f and its n^{th}-order Taylor polynomial P_n centered at a have the same function and derivative values at a, up to order n: i.e., $f^{(k)}(a) = P_n^{(k)}(a)$ for all integers k between 0 and n. If $q(x) = 1 - x + 2\,x^2$ is the second-order Maclaurin polynomial for f, then $f(0) = q(0) = 1$ so we can exclude the left-most plot which shows a curve through the origin. $f'(0) = q'(0) = -1$ so we can exclude the second plot from the left which shows a curve with positive slope as it crosses the y-axis. $f''(0) = q''(0) = 4$ is positive suggesting the graph of f is concave up in some neighborhood of 0, that would lead to rejection of the third plot which is concave down there and choosing the right-most plot which is concave up around $x = 0$.

16. a) $\sin' = \cos$, $\sin'' = -\sin$, $\sin''' = -\cos$, $\sin^{(4)} = \sin$, and $\sin^{(k)} = \sin^{(k-4)}$ for all integers $k \geq 5$. Since $\cos(\pi/2) = 0$, we can infer $\sin^{(\text{odd})}(\pi/2) = 0$; for the even-order derivatives, we have $\sin^{(2\,k)}(\pi/2) = (-1)^k \sin(\pi/2) = (-1)^k$. The fourth-order Taylor polynomial for sin centered at $\pi/2$ is $P_4(x) = 1 - \dfrac{1}{2}\cdot\left(x-\dfrac{\pi}{2}\right)^2 + \dfrac{1}{24}\cdot\left(x-\dfrac{\pi}{2}\right)^4$.

b) The answer to part (a) is a fourth-order Taylor polynomial. The fifth derivative of sin is cos, the maximum value of $|\cos|$ on the interval $[0,\pi]$ is 1. If x is in $[0,\pi]$, then $|x - \pi/2| \leq \pi/2$. Hence the Remainder **Bound** Theorem (11.9.3 on page 677) implies that if x is in $[0,\pi]$, then $\left|\sin(x) - P_4(x)\right| \leq \dfrac{1}{5!}\cdot\left(\dfrac{\pi}{2}\right)^5 = \dfrac{\pi^5}{3840} \approx 0.0797$.

We can get a smaller bound. Since odd-order derivatives of sine have value 0 at $\pi/2$, we can also regard the answer to part (a) as the fifth-order Taylor polynomial and compute the bound $\dfrac{1}{6!}\cdot\left(\dfrac{\pi}{2}\right)^6 = \dfrac{\pi^6}{46080} \approx 0.0209$.

c) $\displaystyle\sum_{k=0}^{\infty}\frac{(-1)^k}{(2\,k)!}\left(x-\frac{\pi}{2}\right)^{2\,k}$ is the Taylor series for $\sin(x)$ centered at $\pi/2$.

d-e) If $x = \pi/2$, then the series in part (c) is $1 + 0 + 0 + \cdots + 0 + \cdots$, clearly convergent. If $x \neq \pi/2$, then none of the terms in that series is zero and their absolute values are positive — therefore the Limit Ratio Test is applicable to that series of those absolute values. The Limit Ratio Test implies the above series is absolutely convergent for all x because

$$\lim_{k\to\infty} \frac{|x-\pi/2|^{2\,(k+1)} \,/\, \big(2\,(k+1)\big)!}{|x-\pi/2|^{2\,k}\,/(2\,k)!} = \lim_{k\to\infty} \frac{|x-\pi/2|^{2}}{(2k+1)\cdot(2k+2)} = 0.$$

Series converges implies individual terms have limit zero, i.e., we now know $\displaystyle\lim_{k\to\infty} \frac{|x-\pi/2|^{2\,k}}{(2\,k)!} = 0$ for all x, hence the Remainder Bound has limit 0 for every x and this series must converge to $\sin(x)$.

17. a) Even-order derivatives of $\sin(x)$ are $\pm\sin(x)$ whose value at $x=0$ is 0; $\sin^{(2k+1)}(x) = (-1)^k \cdot \cos(x)$ whose values at $x=0$ are $(-1)^k$. Hence $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2\,k+1)!} \cdot x^{2\,k+1}$ is the Maclaurin series for $\sin(x)$. The ideas used above in solution for 16 (d-e) are easily adapted to show this series converges to $\sin(x)$ for all x.

b) Odd-order derivatives of $\cos(x)$ are $\pm\sin(x)$ whose value at $x=0$ is 0; $\cos^{(2k)}(x) = (-1)^k\cdot\cos(x)$ whose values at $x=0$ are $(-1)^k$. Hence $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2\,k)!} \cdot x^{2\,k}$ is the Maclaurin series for $\cos(x)$. The ideas used above in solution for 16 (d-e) are easily adapted to show this series converges to $\cos(x)$ for all x.

c) The answers to parts (a) and (b) together with Theorem 11.10.2 (page 687) imply

$$\begin{aligned} \frac{d}{dx}\sin(x) = \frac{d}{dx}\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{(2\,k+1)!}\cdot x^{2\,k+1}\right) &= \sum_{k=0}^{\infty} \frac{d}{dx}\left(\frac{(-1)^k}{(2\,k+1)!}\cdot x^{2\,k+1}\right) \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2\,k+1)!}\cdot(2\,k+1)\cdot x^{2\,k} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2\,k)!}\cdot\frac{2\,k+1}{2\,k+1}\cdot x^{2\,k} \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2\,k)!}\cdot x^{2\,k} = \cos(x). \end{aligned}$$

d) The answers to parts (a) and (b) together with Theorem 11.10.4 (page 689) imply

$$\begin{aligned} \int \sin(x)\,dx = \int\left(\sum_{k=0}^{\infty} \frac{(-1)^k}{(2\,k+1)!}\cdot x^{2\,k+1}\right)dx &= \sum_{k=0}^{\infty}\int\left(\frac{(-1)^k}{(2\,k+1)!}\cdot x^{2\,k+1}\right)dx \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2\,k+1)!}\cdot\frac{x^{2\,k+2}}{2\,k+2} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2\cdot(k+1))!}\cdot x^{2(k+1)} \\ &= \sum_{j=1}^{\infty} \frac{(-1)^{j-1}}{(2\,j)!}\cdot x^{2\,j} = -1\cdot\sum_{j=1}^{\infty}\frac{(-1)^j}{(2\,j)!}\cdot x^{2\,j} \\ &= -1\cdot\left(-1+\sum_{j=0}^{\infty}\frac{(-1)^j}{(2\,j)!}\cdot x^{2\,j}\right) = -\cos(x)+C. \end{aligned}$$

18. Use some facts about geometric series. t in $(-1,1)$ implies $\displaystyle\frac{1}{1+t^2} = \frac{1}{1-(-t^2)} = \sum_{k=0}^{\infty}\left(-t^2\right)^k = \sum_{k=0}^{\infty}(-1)^k\cdot t^{2\,k}$.

If x is in $(-1,1)$, then Theorem 11.10.4 (page 689) implies

$$\arctan(x) = \int_0^x \frac{1}{1+t^2}\,dt = \int_0^x\left(\sum_{k=0}^{\infty}(-1)^k\cdot t^{2\,k}\right)dt = \sum_{k=0}^{\infty}(-1)^k\left(\int_0^x t^{2\,k}\,dt\right) = \sum_{k=0}^{\infty}\frac{(-1)^k}{2\,k+1}\cdot x^{2\,k+1}$$

The fifth-degree term of that Maclaurin series is $(1/5)\cdot x^5$, its fifth-derivative is 24 and that equals $\arctan^{(5)}(0)$.

19. a) $(x-2)^2+y^2 = 2^2$; center at $(2,0)_R$ with radius 2. Plot of circle is shown below in answer to part (c).

b) If $r = 4\,\cos(\theta)$, then $r\,'(\theta) = \dfrac{d}{d\theta}\Big(4\,\cos(\theta)\Big) = -4\,\sin(\theta)$. Then $x = r(\theta)\cdot\cos(\theta)$ and $y = r(\theta)\cdot\sin(\theta)$ imply

$$\frac{dx}{d\theta} = -4\,\sin(\theta)\cdot\cos(\theta) + 4\,\cos(\theta)\cdot\big(-\sin(\theta)\big) = -8\,\sin(\theta)\cdot\cos(\theta) = -4\,\sin(2\,\theta)$$

$$\frac{dy}{d\theta} = -4\,\sin(\theta)\cdot\sin(\theta) + 4\,\cos(\theta)\cdot\cos(\theta) = 4\cdot\Big(\cos(\theta)^2 - \sin(\theta)^2\Big) = 4\,\cos(2\,\theta).$$

If $dx/d\theta \neq 0$, then the point on the curve has a non-vertical tangent whose slope is

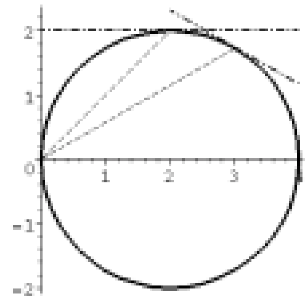
$$\frac{dy/d\theta}{dx/d\theta} = \frac{4 \cdot \left(\cos(\theta)^2 - \sin(\theta)^2\right)}{-8\,\sin(\theta) \cdot \cos(\theta)} = \frac{1}{2} \cdot \frac{\sin(\theta)^2 - \cos(\theta)^2}{\sin(\theta) \cdot \cos(\theta)} = \frac{1}{2} \cdot \left(\tan(\theta) - \cot(\theta)\right) = \frac{1}{2} \cdot \left(\tan(\theta) - \cot(\theta)\right)$$

The alternative expressions for $x'(\theta)$ and $y'(\theta)$ yield a corresponding one for slope:

$$\frac{dy/d\theta}{dx/d\theta} = \frac{4\,\cos(2\,\theta)}{-4\,\sin(2\,\theta)} = -\cot(2\,\theta).$$

c) $\sin(\pi/4) = 1/\sqrt{2} = \cos(\pi/4)$ and the slope of the tangent at the corresponding point is 0. $\sin(\pi/6) = 1/2$, $\cos(\pi/6) = \sqrt{3}/2$ and the slope of the tangent at the corresponding point is $-1/\sqrt{3}$.

The following plot shows the circle, lines through the origin with $\theta = \pi/6$ and $\theta = \pi/4$, together with the lines tangent to the circle at the corresponding points on the circle.



d) The circle is traversed as θ varies in any interval of length π.
(It is acceptable to answer by just mentioning one such interval, e.g., $[0, \pi]$.)

e) Formula 12.2.2 (page 717) implies the length of this closed curve is

$$\int_0^\pi \sqrt{\left(4\cos(\theta)\right)^2 + \left(-4\,\sin(\theta)\right)^2}\,d\theta = \int_0^\pi \sqrt{16\,\cos(\theta)^2 + 16\,\sin(\theta)^2}\,d\theta$$

$$= \int_0^\pi 4 \cdot \sqrt{\cos(\theta)^2 + \sin(\theta)^2}\,d\theta = \int_0^\pi 4 \cdot \sqrt{1}\,d\theta = 4\,\theta\Big|_{\theta=0}^{\pi} = 4\,\pi.$$

f) Formula 12.3.2 (page 721) implies the area of the region enclosed by this curve is

$$\int_0^\pi \frac{1}{2} \cdot \left(4\,\cos(\theta)\right)^2\,d\theta = 4 \cdot \int_0^\pi 2\,\cos(\theta)^2\,d\theta = 4 \cdot \left(\theta + \sin(\theta) \cdot \cos(\theta)\right)\Big|_{\theta=0}^{\pi} = 4\,\pi.$$