Homework #10 Due: Friday May 17

Math 471

Policy: No collaboration. If you have questions, see me.

A cylindrical pipe has a hot fluid flowing through it. Because the pressure is very high, the walls of the pipe are thick. For such a situation, the differential equation that relates temperatures in the metal wall to radial distance is

$$r\,\frac{d^2u}{dr^2}+\frac{du}{dr}=0, \tag{1}$$

where

$$\begin{aligned} r &= \text{radial distance from the centerline,} \\ u &= \text{temperature.} \end{aligned}$$

Consider a pipe with an inner radius of 1 cm and an outer radius of 2 cm containing fluid at 540^oC and an external temperature of 20^oC.

Numerically solve for the temperatures within the pipe by the shooting method and finite differencing under the following conditions.

1. The inner circumference has a temperature equal to the fluid temperature and the outer radius has a temperature equal to the external temperature.
2. Suppose the pipe is insulated to reduce heat loss. The insulation used has the properties such that the gradient du/dr at the outer circumference is proportional to the difference in temperatures from the outer wall to the surroundings:

$$\left.\frac{du}{dr}\right|_{r=2} = 0.083\,[u(2)-20]\,.$$

- Shooting Method: Do the following for each set of conditions.
    1. Hand in the graphs of your first three shots on the same axes. Label them accordingly. State the associated guesses at $u'(1)$ for each solution.
    2. Hand in a plot of the exact solution and your third shot on the same axes and state the maximum absolute error between the approximation and the exact solution over the interval $r\in[1,2]$.
- Finite Differencing: Do the following for each set of conditions.
    1. Make a table of step size versus maximum error for both conditions.
    2. Plot the exact solution and the numerical approximation using a step size that results in a maximum absolute error approximately equal to that of the shooting method. What is this step size?
    3. Hand in a paper copy of the MATLAB code.

Analytic solution: next page →.

Analytic Solution

The differential equation

$$r\,\frac{d^2u}{dr^2}+\frac{du}{dr}=0,$$

has an exact solution of the form

$$u(r)=C_1\ln(r)+C_2.$$

C_1 and C_2 may be determined from the boundary conditions.