Homework #3 Due: Friday February 22

Math 471

Problem 1 should be written up and given to me. Problem 2 must be demonstrated in class on Friday February 22.

1. Accelerating linear convergence with **Aitken's Method**.

   Assume that the sequence $\{x_n\}_{n=0}^{\infty}$ converges linearly to the value r and that $x_n - r \neq 0$ for all $n \geq 0$. If there exist a real number A where $0 < |A| < 1$ such that

   $$\lim_{n\to\infty} \frac{r - x_{n+1}}{r - x_n} = A,$$

   then the sequence $\{q_n\}_{n=0}^{\infty}$ defined by

   $$q_n = x_n - \frac{(x_{n+1} - x_n)^2}{x_{n+2} - 2x_{n+1} + x_n} \tag{1}$$

   converges to r faster than $\{x_n\}_{n=0}^{\infty}$, in the sense that

   $$\lim_{n\to\infty} \frac{r - q_n}{r - x_n} = 0 \tag{2}$$

   Assignment: Prove equation (2)

   Hint Derivation of Equation (1)

   $$\frac{r - x_{n+1}}{r - x_n} \approx A \quad \text{and} \quad \frac{r - x_{n+2}}{r - x_{n+1}} \approx A \quad \text{for } n \text{ large}$$

   therefore

   $$(r - x_{n+1})^2 \approx (r - x_n)(r - x_{n+2})$$

   and solving for r you get

   $$r \approx \frac{x_{n+2}x_n - x_{n+1}^2}{x_{n+2} - 2x_{n+1} + x_n} = q_n$$

   which is equivalent to the right hand side of (1)

2. Finding Roots of a Cubic polynomial.

   Write a program that prompts the user for the coefficients of a cubic polynomial. Use Newton's method to find one root of the polynomial, deflate the polynomial and then use the improved quadratic formula to find the remaining two roots (which may be complex). If the remaining two roots are real, use Newton's method to polish these with the original cubic.