

Homework #5

Due: Friday March 6
Math 471

Consider the system of linear equations defined by $Ax = b$ where

$$A = \begin{pmatrix} -4 & 1 & 1 & 1 \\ 1 & -4 & 1 & 1 \\ 1 & 1 & -4 & 1 \\ 1 & 1 & 1 & -4 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

Solve this system using the Gauss-Seidel method with Over-relaxation, starting with an initial guess of $x = [0, 0, 0, 0]^T$, a tolerance on the 2-norm of the residual of 10^{-5} , and an initial value of $\omega = 1$. You may find it useful to download the program GS.m from the website.

Correction: Not all values of ω between 1 and 2 will be an improvement on $\omega = 1$. I stated in class that any ω greater than 1 less than 2 would speed up convergence. This is clearly false as your results should show.

1. Determine the optimal over-relaxation factor ω . Compare your results with those displayed in Table 2.2 of the text. If the number of iterations remains a minimum for many values of ω , state a range of optimal ω 's. See if you can find an optimal $\omega \neq 1.3$. Provide evidence verifying your conclusion such as a table or a graph.
2. See if altering any of the following features of the problem effects the value of your optimal ω . Choose at least two of the following. Describe the changes you made and the resulting change in the optimal ω (if any).
 - The size of A (Make sure A remains diagonally dominant but maintains a similar form.)
 - The tolerance level
 - The initial guess at the solution
 - The b -vector
 - The type of norm used on the residual vector

Both of these should be written out with any accompanying graphs or tables included. Please be clear in your write-up.