

**Homework #7** Due: Friday April 12  
Math 471

1. Derive a general formula for the optimal step size ( $h_{opt}$ ) using  $\mathbf{O}(h^4)$  central differencing and  $\mathbf{O}(h)$  forward differencing approximations to the derivative.
2. Demonstrate that the above formulas are valid by plotting the  $\log(h)$  -vs-  $\log(\text{error})$  for the first derivative approximations of  $f(x) = e^x$  evaluated at  $x_i = 1$ . Set  $\varepsilon$  (the maximum round-off error) to be  $10^{-15}$ .
3. What is the slope of these curves before round-off error starts adversely effecting the approximation and why is this appropriate?
4. Compare your results to those from the example using order  $h^2$  central differencing. What are the practical benefits and drawbacks of the different schemes.