

## Homework #8 Due: Friday April 19

Math 471

1. Cardiac Output may be approximated by Stretching or Compressing the function

$$Q(t) = \sin^n(\pi t) \cos(\pi t - \phi)$$

where  $n = 13$  and  $\phi = \pi/10$ . Figure 1 shows a graph of this function, where the flow is in ml/(unit time). One value of particular interest to cardiologists is the peak-to-mean flow ratio. To calculate this from the approximating function  $Q(t)$ , you need first, to find the time ( $t^*$ ) at which the peak outflow occurs. We did this in homework 1 and found that  $t^* = 0.4517$ .

Now, we need to calculate the mean flow by

$$Q_{ave} = \int_0^1 Q(t) dt. \tag{1}$$

The peak-to-mean flow ratio is then given by

$$\text{peak-to-mean flow ratio} = \frac{Q(t^*)}{Q_{ave}} \tag{2}$$

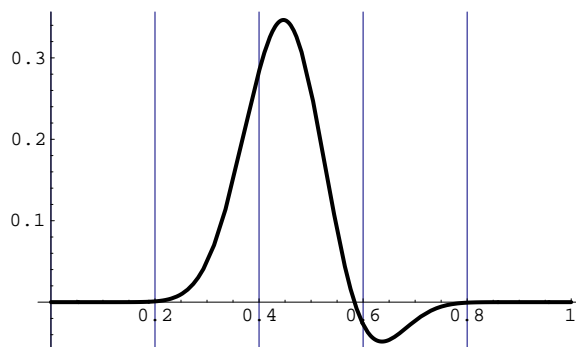


Figure 1:  $Q(t)$

**Assignment:** Find the peak to mean flow ratio based on numerically integrating the definite integral for  $Q_{ave}$ . You may use any of the composite Newton-Cotes formulas (except the trapezoid rule) or Gaussian Quadrature. You may also interpolate points with a cubic spline and then integrate this. In any of these methods make sure that  $h \leq 0.1$ .

If you choose cubic spline interpolation, MATLAB can do most of this for you but you have to figure out how to do it.

This can be done symbolically and here are the results from Mathematica.

$$Q_{ave} = 0.0647306$$

$$\text{peak-to-mean flow ratio} = \frac{Q(t^*)}{Q_{ave}} = 5.97182$$

**Grad Students:** Find  $A_n$  and  $B_n$  for  $0 \leq n \leq 8$  in the Fourier series representation of  $Q(t)$ .