

Homework #9 Due: Friday May 3

Math 471

1. **Stability of Modified Euler's Method:** Consider the following IVP to be solved by the modified Euler method:

$$y' = \lambda y \quad y(x_0) = y_0 \neq 0$$

Show that the stability requirement for modified Euler's method is identical to that of Euler's method. The only difference is in the form of the instability that occurs. Describe and justify the difference in the instability properties between these two methods.

2. Cardiac Output may be approximated by the function

$$Q(t) = A \sin^n(\omega t) \cos(\omega t - \phi) \tag{1}$$

where $n = 13$, $\phi = \pi/10$, $\omega = 76\pi$ and $A = 106,596$. Figure 1 shows a graph of this function, where the flow is in ml/min and time is in minutes. $p = 1/76$.

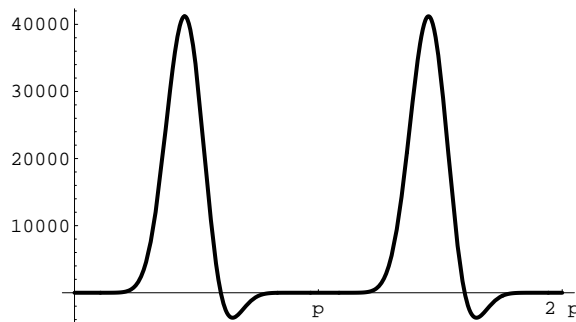


Figure 1: $Q(t)$

A model for Arterial pressure P_A and Venous pressure P_V in mmHg, starts with a few assumptions and applies two conservation equations, (the details are on the second page), and results in the following system of differential equations for pressures.

$$\frac{dP_A}{dt} = \frac{1}{C_A} \left(-\frac{1}{R}P_A + \frac{1}{R}P_V + Q(t) \right) \quad P_A(0) = \bar{P}_A \tag{2}$$

$$\frac{dP_V}{dt} = \frac{1}{C_V} \left(\frac{1}{R}P_A - \frac{1}{R}P_V - Q(t) \right) \quad P_V(0) = \bar{P}_V \tag{3}$$

where

$$R = \frac{\bar{P}_A - \bar{P}_V}{\bar{Q}} \quad \bar{P}_A = 96 \quad \bar{P}_V = 5 \quad \text{and} \quad \bar{Q} = 6900$$

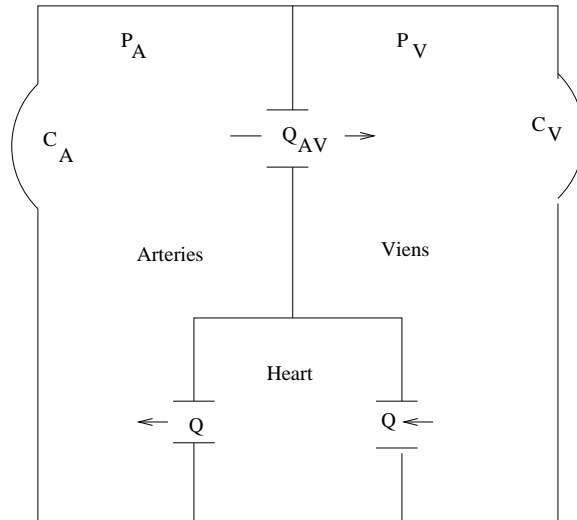
$$C_A = 2 \quad C_V = 55,$$

and $Q(t)$ is from equation 1.

Your Assignment Solve this system of differential equations using MATLAB's ode45 solver over a time period of $t = 0 \rightarrow 10/76$ min. This corresponds to 10 cardiac cycles.

Hand in a plot of P_A . What is the max P_A and the minimum P_A ? These correspond to systolic and diastolic blood pressures.

A MODEL FOR ARTERIAL AND VENOUS BLOOD PRESSURES



Derivation of the Governing Differential Equations

- Assumption 1:

The flow between Arteries and Veins Q_{AV} is proportional to the pressure difference.

$$Q_{AV} = \frac{P_A - P_V}{R}$$

In this equation, R is called the resistance to flow.

- Assumption 2:

Volume changes are proportional to the pressure changes:

$$\frac{dV_A}{dt} = C_A \frac{dP_A}{dt} \quad (4)$$

$$\frac{dV_V}{dt} = C_V \frac{dP_V}{dt} \quad (5)$$

(6)

In these equations, C_A and C_V are the compliances.

- Conservation of mass in each compartment

$$\text{Flow Rate in} - \text{Flow Rate out} = \text{rate of volume change} \quad (7)$$

$$Q(t) - \frac{P_A - P_V}{R} = C_A \frac{dP_A}{dt} \quad (8)$$

$$\frac{P_A - P_V}{R} - Q(t) = C_V \frac{dP_V}{dt} \quad (9)$$