

Math 22, Section 5, Sample Final

Our Final is on Thursday, May 6 at 2:00 in this room.

Show all relevant work for full credit.

1. Sketch the graph of the following functions clearly labelling all intercepts (if any) and asymptotes (if any). If the graph is a parabola give the vertex as well.

(a) $f(x) = 3(x - 2) + 5$

(b) $f(x) = x^2 - x + 6$

(c) $f(x) = x^3 - 2x^2 - 3x$

(d) $f(x) = \frac{2x - 2}{x + 2}$

(e) $f(x) = 7e^{-x} + 1$

(f) $f(x) = -\ln(x + 2)$

2. Find the equation of the line with the following properties. Put your answer in slope intercept form: $y = mx + b$.

(a) The line has slope $3/2$ and goes through the point $(2,5)$.

(b) The line goes through the points $(0,4)$ and $(5,-2)$.

3. Linear Modelling

(a) On a yardstick you notice that 13 inches is the same length as 33 centimeters. Use this information to find a mathematical model (formula) that expresses centimeters in terms of inches.

(b) You have one of those cars that gives your speed in miles per hour (m/h) and in kilometers per hour (Km/h). At some point you notice that you are travelling 64 miles per hour and this is also 103 Km/hour. Find a formula that relates miles to kilometers.

(c) Recall that water freezes at 0° Celsius (32° Fahrenheit) and boils at 100° Celsius (212° Fahrenheit). Use this information to find the equation of the line that gives the relationship between temperatures in Celsius (C) and Fahrenheit (F).

4. Quadratic Modelling. You must justify your answer with relevant work.

(a) A rancher has 40 feet of fencing with which to make a rectangular rabbit pen. He can use the barn as one side of the pen. What measurements will produce the maximum enclosed area.

(b) A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals. What measurements will produce the maximum enclosed area?

(c) A manufacturer of lighting fixtures has a daily production cost of

$$C(x) = 800 - 10x + x^2$$

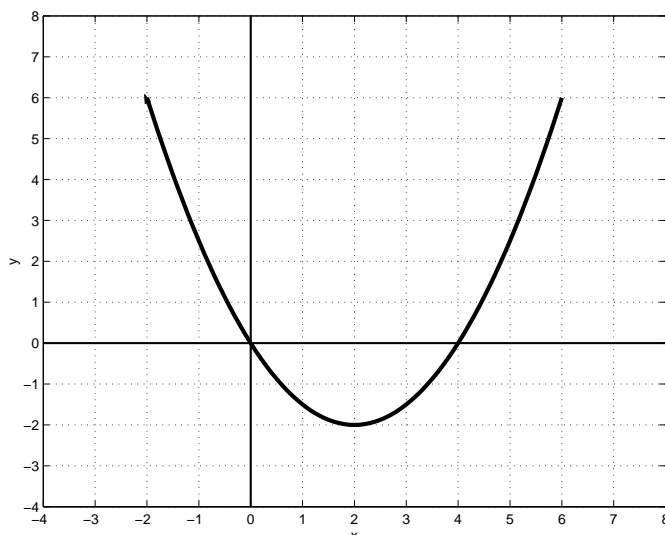
where C is the total cost in dollars and x is the number of units produced. How many fixtures should be produced each day to yield a minimum cost?

5. Decide whether the equation represents y as a function of x . If it does, state the function.

(a) $x^2 - 2x + y^2 = 4$

(b) $x^3 + 4y = 8$

6. Consider the graph of a function $f(x)$ pictured below.



Sketch the graph of the functions indicated.

(a) $g(x) = f(x + 2)$

(b) $h(x) = -f(x) + 3$

(c) $q(x) = f(-x)$

7. Consider the two functions below

$$f(x) = x^2 \quad \text{and} \quad g(x) = x + 1$$

recall that $(f \circ g)(x) = f(g(x))$.

(a) Find $(f \circ g)(2)$.

(b) Find $(g \circ f)(2)$.

8. Determine whether the function has an inverse. If it does, find it.

(a) $f(x) = 5 - x^2$

(b) $f(x) = \sqrt{3x - 6}$

(c) $f(x) = 2x - 3$

9. Find a polynomial function of degree 3 which has a graph with x -intercepts at $x = -1$, 0 , and $3/2$.

10. The graph of the function $f(x) = x^3 - 12x - 16$ has an x -intercept at $x = -2$. Use this information to completely factor this polynomial. What are the zeros of this polynomial?

11. The function $f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$ has zeros at $x = 2$ and at $x = -3$. Use this information to completely factor this polynomial.

12. The polynomial function $f(x) = x^3 - 28x - 48$ has the property that $f(-4) = 0$. Use this information to completely factor the polynomial function. What are the zeros of this polynomial?

13. Express the following as the logarithm of single quantity.

(a) $\log_3(x) + \log_3(5)$

(b) $5 \ln(x) - 2 \ln(3)$

14. Given that $\log_a(x) = 6$, $\log_a(y) = 2$ and $\log_a(z) = 3$ find

(a) $\log_a\left(\frac{xy}{z}\right)$

(b) $\log_a(\sqrt{x})$

15. Solve for x . **Give exact answers**, not decimal approximations.

(a) $3 = \log_2(x)$

(b) $x = \ln e^{\sqrt{5}}$

(c) $\frac{4}{1 + e^x} = 1$

(d) $\ln(x) = 5 + \ln(3)$

(e) $e^{\ln \sqrt{2}} = x$

(f) $e^x = 5 - 7e^x$

(g) $25e^{2x+1} = 100$

(h) $\ln(x) + \ln(x - 2) = 1$

16. Exponential Growth and Decay

(a) You invest 2000 dollars in an account earning r percent interest compounded continuously. After 4 years there is 3000 dollars in the account. Find the exact value of r , not a decimal approximation.

(b) You invest 100 dollars in an account earning 7% interest compounded continuously. How long will it take until you have 150 dollars in the account. Give the exact value.

(c) Radioactive radium has a half life of 1620 years.

i. How long does it take for a given amount of radioactive radium to decrease by one half? Exact answer.

ii. How long does it take for an initial amount of radium to decrease to one tenth the original amount. Exact answer.

17. A conservation organization releases 100 animals into a game preserve. They believe that the preserve has a carrying capacity of 1000 animals and that the growth of the herd will be modeled by the logistic curve

$$P = \frac{1000}{1 + 9e^{-kt}} \quad t \geq 0$$

where P is the number of animals and t is the time in years. Find k if the herd size is 200 after 2 years.

New Material: Chapter 6.3 (Linear Systems) and Appendix B.1 (Conic Sections)

18. Solve the following systems. In the case of infinitely many solutions put your answer in the form $z = a$, $y = f(a)$, and $x = g(a)$. As a guide I have put in parentheses the type of solution. This will not appear on the final.

(a) (Unique Solution)

$$\begin{aligned}x + y + z &= 3 \\4x - y + 3z &= 7 \\5y + 2z &= 0\end{aligned}$$

(b) (Unique Solution)

$$\begin{aligned}4x + y - 3z &= 11 \\2x - 3y + 2z &= 9 \\x + y + z &= -3\end{aligned}$$

(c) (Unique Solution)

$$\begin{aligned}3x + 2z &= 13 \\x + 2y &= -5 \\-3y - z &= 10\end{aligned}$$

(d) (Infinitely many solutions)

$$\begin{aligned}x + 2y - z &= 5 \\4x - y + 5z &= 11 \\5x - 8y + 13z &= 7\end{aligned}$$

(e) (No solutions)

$$\begin{aligned}3x - 2y - 6z &= -4 \\-3x + 2y + 6z &= 1 \\x - y - 5z &= -3\end{aligned}$$

19. State whether the equation describes a line, parabola, circle, ellipse, or hyperbola. Then sketch the graph of the equation.

(a) $3x + 2y = 4$

(b) $y^2 - 8x = 0$

(c) $y = x^2 - 3x - 4$

(d) $\frac{x^2}{16} + \frac{y^2}{25} = 1$

(e) $\frac{x^2}{16} - \frac{y^2}{25} = 1$

(f) $x^2 - 2x + y^2 = 16$

20. Find the vertex and focus of the parabola and sketch its graph.

$$y^2 = -6x$$

21. Find an equation of the parabola with vertex at the origin and focus at $(0,-2)$.

22. Find the center and vertices of the ellipse and sketch its graph.

$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$

23. Find an equation of the ellipse with center at the origin with vertices at $(2,0)$ and $(-2,0)$ and with a minor axis of length 3.