

Math 231, Section 2, FINAL EXAM

Name:(10 pts)

Graphing calculators are allowed during this exam, **except TI-89's and TI-92's**. There is no penalty for guessing on the true-false and multiple choice questions. Indicate your answers **CLEARLY** and **NEATLY**.

1. If \mathbf{u} is a vector of length 8, \mathbf{v} is a vector of length 3, and the cosine of the angle between \mathbf{u} and \mathbf{v} is $\frac{1}{4}$, then the value of $\mathbf{u} \cdot \mathbf{v}$ is:

(5 pts)

- (a) 4 (b) 6 (c) 20 (d) 28 (e) 96

2. If $\mathbf{v} = 2\mathbf{i} + 6\mathbf{j} + 9\mathbf{k}$ and $\mathbf{u} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, then the length of the projection of \mathbf{u} in the direction of \mathbf{v} is:

(5 pts)

- (a) $\frac{16}{3}$ (b) $\frac{28}{3}$ (c) $\frac{16}{11}$ (d) $\frac{28}{11}$ (e) $\frac{16}{33}$

3. A normal vector to the plane containing the points (1,0,1), (1,-1,2), and (2,1,0) is:

(5 pts)

- (a) $\mathbf{j} + \mathbf{k}$ (b) $-\mathbf{j} + \mathbf{k}$ (c) $-\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ (d) $\mathbf{i} - \mathbf{j} - \mathbf{k}$ (e) $-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

4. Suppose that at time $t = 1$ the motion of a particle along a path is such that its velocity is $\mathbf{v}(1) = -2\mathbf{j}$ and its acceleration is $\mathbf{a}(1) = 4\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$. Then the curvature at $t = 1$ is

(5 pts)

- (a) $1/5$ (b) $4/5$ (c) $5/4$ (d) 4 (e) 5

5. The direction in which the function $f(x, y) = xy^2 + x^2y$ is decreasing most rapidly at the point $(2, -1)$ is:

(5 pts)

- (a) $-2\mathbf{i} + \mathbf{k}$ (b) $\frac{\mathbf{i} + \mathbf{j}}{\sqrt{2}}$ (c) $\mathbf{i} + \mathbf{j}$ (d) $-\mathbf{i}$ (e) \mathbf{i}

6. The rate of change of $f(x, y) = x^2y$ at the point $(5, 2)$ in the direction of the vector $4\mathbf{i} + 3\mathbf{j}$ is:

(5 pts)

- (a) 5 (b) 23 (c) 31 (d) 155 (e) $16\mathbf{i} + 15\mathbf{j}$

7. The equation of the tangent plane to the surface $xyz = -6$ at the point $(2, -1, 3)$ is:

(5 pts)

(a) $2x - y + 3z = -6$

(b) $2x - y + 3z = 14$

(c) $x + y + z = 0$

(d) $3x - 6y + 2z = 18$

(e) $8x + y - 9z = -12$

8. Suppose that $z = \sin xy$ and that x and y are functions of t . Suppose that when $t = 1$ we have

$$x = 2, \quad y = 3, \quad \frac{dx}{dt} = 5, \quad \frac{dy}{dt} = 7.$$

Then when $t = 1$, $\frac{dz}{dt} =$

(5 pts)

(a) $29 \cos(6)$

(b) $210 \cos^2(6)$

(c) $12 \cos(6)$

(d) $31 \cos(6)$

(e) $\cos(6)$

9. Find the arc length of the circular helix $\mathbf{r}(t) = (\cos(t), \sin(t), t)$ from $t = 0$ to $t = 2\pi$.

(10 pts)

your answer:

10. The position vector of a particle at time t is given by

(20 pts)

$$\mathbf{r}(t) = 2 \cos(t) \mathbf{i} - \sin(t) \mathbf{j} + t \mathbf{k}$$

(a) Determine the velocity at time $t = \pi$.

your answer:

(b) Determine the speed at time $t = \pi$.

your answer:

(c) Determine the acceleration at time $t = \pi$.

your answer:

(d) Determine the unit tangent vector \mathbf{T} at time $t = \pi$.

your answer:

11. Find and classify all of the critical points of $f(x, y) = x^3 + y^2 - 6x^2 - 2y$.

(20 pts)