

**Math 231, Section 1, TEST 2 (Chapter 11) Name:** .....(10 pts)

Graphing calculators are allowed during this exam, **except TI-89's and TI-92's**. There is no penalty for guessing on the true-false and multiple choice questions. Indicate your answers CLEARLY and NEATLY.

1. Consider a smooth curve  $C$  represented by the vector function  $\mathbf{r}(t)$ .

(10 pts)

- (a) **T**    **F**    If the curvature at a point  $P$  is 4, then the radius of curvature is 0.25.
- (b) **T**    **F**    The acceleration vector is always perpendicular to the unit tangent vector.
- (c) **T**    **F**    The acceleration vector lies in the same plane as the unit tangent vector and the unit normal vector.
- (d) **T**    **F**    The velocity vector is parallel to the unit tangent vector.
- (e) **T**    **F**    If  $\mathbf{r}(t)$  represents position and  $s(t)$  represents the associated arc length, then  $\frac{ds}{dt} = \mathbf{r}'(t)$ .

2. The principal unit normal vector to the graph of  $y = \sin x$  at the point  $(\pi/2, 1)$  is

(5 pts)

- (a)  $\langle 1, 0 \rangle$       (b)  $\langle 0, 1 \rangle$       (c)  $\langle -1, 0 \rangle$       (d)  $\langle 0, -1 \rangle$

3. Represent the plane curve  $y = x^2 + 2$  by a vector valued function  $\mathbf{r}(t)$ .

(5 pts)

your answer:
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4. Sketch the graph of the vector valued function  $\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j}$  for  $0 \leq t \leq 2\pi$ .

(5 pts)

5. The position vector of a particle at time  $t$  is given by

(40 pts)

$$\mathbf{r}(t) = \cos(2t) \mathbf{i} + \sin(2t) \mathbf{j} + t \mathbf{k}$$

(a) Determine the velocity at time  $t = \pi$ .

your answer:

(b) Determine the speed at time  $t = \pi$ .

your answer:

(c) Determine the acceleration at time  $t = \pi$ .

your answer:

(d) Determine the unit tangent vector  $\mathbf{T}$  at time  $t = \pi$ .

your answer:

(e) Determine the principal unit normal vector  $\mathbf{N}$  at  $t = \pi$ .

your answer:

(f) Determine the tangential component of acceleration at  $t = \pi$ .

your answer:

(g) Determine the normal component of acceleration at  $t = \pi$ .

your answer:

(h) Determine the curvature of the path at time  $t = \pi$ .

your answer:

6. A baseball is hit 3 feet above the ground at 128 feet per second and at an angle of  $\frac{\pi}{6}$  with respect to the ground. Assume that the only force acting on the ball after it is hit is that due to gravity. ( $g = 32$  feet/(sec)<sup>2</sup>).

(15 pts)

- (a) Use the fact that acceleration is constant and given by  $\mathbf{a}(t) = -g \mathbf{j}$  to derive a function for the position of the ball  $\mathbf{r}(t)$  for any time  $t$ . Show your derivation below and write the result in the box.

your answer:

- (b) What is the maximum height the ball reaches?

your answer:

7. The DNA molecule has the shape of a double helix. The radius of each helix is about 10 angstroms (1 angstrom =  $10^{-8}$  cm). Each helix rises about 34 angstroms during each complete turn and there are about  $2.9 \times 10^8$  complete turns, so the vector valued function defining each helix is (10 pts)

$$\mathbf{r}(t) = (10 \cos t)\mathbf{i} + (10 \sin t)\mathbf{j} + \left(\frac{34}{2\pi}t\right)\mathbf{k}, \quad 0 \leq t \leq 2\pi \times 10^8.$$

Determine the length of the helix.

your answer: