Math 231, Section 2, TEST 3 (Chapter 12) Name: .....(*10 pts*)
Graphing calculators are allowed during this exam, **except TI-89's and TI-92's**. There is no penalty for guessing on the true-false and multiple choice questions. Indicate your answers CLEARLY and NEATLY.

1. Consider a function $f(x, y)$ that is differentiable at all (x, y).

(*10 pts*)

(a) **T** **F** If $\nabla f \neq 0$, then ∇f is orthogonal to the level curves of f.

(b) **T** **F** If $\mathbf{u} =< 0, 1 >$ then $D_{\mathbf{u}} f(x, y) = f_x(x, y)$.

(c) **T** **F** If $\mathbf{u} =< 1, 0 >$ then $D_{\mathbf{u}} f(x, y) = \lim_{\Delta x \to 0} \dfrac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$.

(d) **T** **F** If $\nabla f(x_o, y_o) = 0$, then (x_o, y_o) is a critical point.

(e) **T** **F** The direction of steepest **descent** of f is given by ∇f.

2. The equation of the tangent plane to the surface $xy^2z^3 = 12$ at the point (3,2,1) is:

(*10 pts*)

(a) $3x + 2y + z = 14$ (b) $x + 2y + 3z = 10$ (c) $2x + 3y + 12z = 24$

(d) $x + 3y + 9z = 18$ (e) $x + 3y = 4$

3. If $f(x, y) = x^2y^2 + ax^2y + bxy^2 + cxy + d$ then $f_{xy} =$

(*10 pts*)

(a) $2xy^2 + 2axy + by^2 + cy$ (b) $2x^2y + ax^2 + 2bxy + cx$ (c) $2y^2 + 2ay$

(d) $2x^2 + 2bx$ (e) $4xy + 2ax + 2by + c$

4. If $\mathbf{v} =< 3, 4 >$ and $f(x, y) = x^2 + y^2$, then the directional derivative of f in the direction of $\mathbf{v}$ at the point (1,1) is

(*10 pts*)

(a) 70 (b) $\dfrac{14}{5}$ (c) $2\mathbf{i} + 2\mathbf{j}$ (d) $10\sqrt{2}$ (e) 5

5. Given $z = x^2 + y^3$ use the total differential to estimate the change in z as (x, y) goes from (1,1) to (1.1,1.2). *(10 pts)*

your answer:

6. Given $w = xyz$, where $x = t^2$, $y = 2t$, and $z = e^{-t}$. Find dw/dt using the chain rule. *(10 pts)*

your answer:

7. Find a unit normal vector to the surface $x^2 + y^2 + z^2 = 11$ at the point (3,1,1). *(10 pts)*

your answer:

8. Let $g(x, y) = x^2 - 3xy - y^2$. Find the critical point(s) of g and determine whether there is a relative minimum, relative maximum, or saddle point at each. *(10 pts)*

your answer:

9. A rectangular box **WITH NO TOP** is to be constructed which has a volume of 32 cubic feet. Find the dimensions of the box which minimize the surface area. I suggest you **DO NOT** use Langrange Multipliers. *(10 pts)*

your answer: