Sample Exam 1 **For Math 230 sections 1 and 2**

The actual exam will not be this long and WILL include some questions not represented here.

1. True-False

   (a) **T** **F** The vectors $< 2, -1, 3 >$ and $< 4, -2, 6 >$ are parallel.

   (b) **T** **F** The vectors $< 1, 2, 3 >$ and $< 1, -5, 3 >$ are orthogonal.

   (c) **T** **F** The vectors $< 4, 1, 2 >$ and $< -2, 4, 7 >$ have the same length.

   (d) **T** **F** The vector $< 1, 1, 0 >$ is a unit vector.

   (e) **T** **F** The vector $< 2, 1, -1 >$ is normal (perpendicular) to the plane $2x + y - z = 3$.

   (f) **T** **F** If $\mathbf{w} = \mathbf{u} \times \mathbf{v}$, then $\mathbf{w}$ is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

   (g) **T** **F** If $\mathbf{w} = 2\,\mathbf{u}$, then $||\mathbf{u} \times \mathbf{v}|| = 2\,||\mathbf{w} \times \mathbf{v}||$.

2. If $\mathbf{u}$ has length 3, $\mathbf{v}$ has length 2, and the angle between $\mathbf{u}$ and $\mathbf{v}$ is 60^o, then $\mathbf{u} \cdot \mathbf{v} =$

   (a) $1/2$ (b) 6 (c) 3 (d) $3/2$ (e) $3\sqrt{3}$

3. If $\mathbf{u} = 2\mathbf{i} + 2\mathbf{j}$ and $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, then $\text{proj}_{\mathbf{v}}\mathbf{u}$ (the projection of $\mathbf{u}$ onto $\mathbf{v}$) is

   (a) $\frac{4}{3}\,\mathbf{u}$ (b) $\frac{4}{3}\,\mathbf{v}$ (c) $2\,\mathbf{v}$ (d) 2 (e) $\sqrt{2}\,\mathbf{v}$

4. If $\mathbf{u} = \langle 3, 0, 4 \rangle$ then the unit vector in the direction of $\mathbf{u}$ is

   (a) $\left\langle \frac{3}{5}, 0, \frac{1}{5} \right\rangle$ (b) $\left\langle \frac{5}{3}, 0, \frac{5}{3} \right\rangle$ (c) $\left\langle \frac{1}{2}, 1, \frac{2}{3} \right\rangle$ (d) 5 (e) $\left\langle \frac{3}{5}, 0, \frac{4}{5} \right\rangle$

5. The surface whose equation in *cylindrical* coordinates is given by $r = 3$ is

   (a) a cone (b) a cylinder (c) a sphere (d) a plane (e) an ellipsoid

6. In cylindrical coordinates the point $(r, \theta, z) = (4, \pi/6, 3)$. This point in rectangular (cartesian) coordinates is

   (a) $(\sqrt{2}/2, \sqrt{2}/2, 3)$ (b) $(2, 1/2, 3)$ (c) $(2\sqrt{3}, 2, 3)$ (d) $(2\sqrt{3}, 2, 5)$ (e) $(2\sqrt{3}, 2, 9)$

7. The point of intersection of the line given by $\frac{x+2}{2} = \frac{y-1}{8} = z + 2$ and the plane y = 9 is

   (a) $(3, 2, 1)$ (b) $(5, 9, 7)$ (c) $(1, 9, 5)$ (d) $(0, 9, -1)$ (e) $(1, 2, 5)$

8. Let $\mathbf{u} = \sqrt{2}\,\mathbf{j} - \mathbf{k}$, $\mathbf{v} = \sqrt{2}\,\mathbf{i} + \mathbf{k}$ and θ be the angle between these vectors. In this case, $\cos(\theta)$ is

   (a) $\frac{-1}{3}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{-1}{\sqrt{2}}$ (d) $\frac{1}{3}$ (e) $\sqrt{2}$

9. If $\mathbf{u} = \langle 1, -1 \rangle$ and $\mathbf{v} = \langle -1, 2 \rangle$ then $||\mathbf{u} + \mathbf{v}||$ is
   (a) $\sqrt{13}$ (b) 1 (c) $\sqrt{5}$ (d) 3 (e) $\sqrt{2}$

10. A vector $\mathbf{v}$ has magnitude equal to 2 and makes an angle of -45^{o}with the positive x-axis. This vector in component form is then
    (a) $\left\langle \frac{-1}{2}, \frac{1}{2} \right\rangle$ (b) $\langle 2, -2 \rangle$ (c) $\left\langle -\sqrt{2}, \sqrt{2} \right\rangle$ (d) $\left\langle \sqrt{2}, -\sqrt{2} \right\rangle$ (e) $\left\langle \frac{1}{2}, \frac{1}{2} \right\rangle$

11. Let $\mathbf{u} = \langle 1, 0, 2 \rangle$ and $\mathbf{v} = \langle 1, 1, 3 \rangle$. Find a **unit** vector that is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$.

12. Let P(1,3,2), Q(3,1,1), R(-1,-2,3) be three points in $\mathbf{R}^3$.
    (a) Find the vector (in component form) from P to Q.
    (b) Find the parametric equations for the line in space through points P and Q. Parametric equations describe x, y, and z in terms of a parameter, usually t.
    (c) Find a vector (in component form) that is orthogonal to $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.
    (d) Find an equation for the plane determined by the points P,Q, and R.

13. Consider the surface defined by $z - \dfrac{x^2}{4} - y^2 = 0$.
    (a) Sketch the trace of the surface in the plane $z = 4$. Label the axes and clearly indicate at least two points on the trace.
    (b) Sketch the trace of the surface in the yz-plane. Label the axes and clearly indicate at least two points on the trace.
    (c) Sketch the trace of the surface in the xz-plane. Label the axes and clearly indicate at least two points on the trace.
    (d) Sketch the surface in 3-space. Label the axes.

14. Consider the vector $\mathbf{u}$ in the yz-plane of length 4 making an angle of 30^o with the positive y-axis.
    (a) Write the vector $\mathbf{u}$ in standard unit vector notation (as a linear combination of $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$).
    (b) Write the vector in component form.
    (c) Sketch the vector $\mathbf{u}$.

15. Consider the surface in 3-space defined by the equation $x^2 + y^2 = 4y$.
    (a) Sketch and describe the surface in 3-space.
    (b) Convert the equation into cylindrical coordinates.

16. **Bonus** Find the point (x_1, y_1, z_1) that results when the point (x_o, y_o, z_o) is projected onto the plane $ax + by + cz + d = 0$.