Test 2 **Sample**

The actual exam will not be this long and WILL include some questions not represented here.

1. Consider a smooth curve C represented by the vector function $\mathbf{r}(t)$.

   (a) **T** **F** The unit normal vector is always orthogonal to the velocity vector.

   (b) **T** **F** The acceleration vector is always orthogonal to the unit tangent vector.

   (c) **T** **F** $D_t[r(t) \cdot r(t)] = 2\, r'(t) \cdot r(t)$

   (d) **T** **F** If $||\mathbf{v}(t)||$ is constant then $\mathbf{a}(t)$ is orthogonal to $\mathbf{v}(t)$.

   (e) **T** **F** If $\mathbf{r}(t)$ represents position and $s(t)$ represents the associated arc length, then $\dfrac{ds}{dt} = ||\mathbf{r}'(t)||$.

   (f) **T** **F** If the curvature at a point P is 4, then the radius of curvature is 0.25.

   (g) **T** **F** The acceleration vector is always perpendicular to the unit tangent vector.

   (h) **T** **F** The acceleration vector lies in the same plane as the unit tangent vector and the unit normal vector.

   (i) **T** **F** The velocity vector is parallel to the unit tangent vector.

   (j) **T** **F** If $\mathbf{r}(t)$ represents position and $s(t)$ represents the associated arc length, then $\dfrac{ds}{dt} = \mathbf{r}'(t)$.

2. The principal unit normal vector to the graph of $y = \cos x$ at the point $(\pi, -1)$ is
   (a) $< 1, 0 >$ (b) $< 0, -1 >$ (c) $< -1, 0 >$ (d) $< 0, 1 >$

3. Consider a curve in space generated by $\mathbf{r}(t) = (2t + 1)\,\mathbf{i} - 3t\,\mathbf{j} + 10\,\mathbf{k}$.

   The length of this curve as t goes from 0 to 2 is
   (a) $\sqrt{13}$ (b) $4\sqrt{13}$ (c) $2\sqrt{5}$ (d) $2\sqrt{13}$ (e) $\sqrt{5}$

4. The position vector of a particle at time t is given by $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + 2t\,\mathbf{k}$

   The speed of the particle at time $t = 1$ is
   (a) $\sqrt{13}$ (b) 3 (c) $\sqrt{10}$ (d) $2\sqrt{10}$ (e) 5

5. A baseball is hit 3 feet above the ground at 128 feet per second and at an angle of $\frac{\pi}{6}$ with respect to the ground. Assume that the only force acting on the ball after it is hit is that due to gravity. ($g = 32$ feet/(sec)^2). What is the height of the ball at $t = 2$ seconds? Answers are in feet.
   (a) 67 (b) 62 (c) 32 (d) 48 (e) 128

6. The curve generated by $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}$ has the same shape as the curve
   (a) $x = y^2$ (b) $y = x^2$ (c) $y = \sqrt{x}$ (d) $y = 2x$ (e) $y = x$

7. Find $\mathbf{r}(t)$ given $\mathbf{r}'(t) = e^{-t}\mathbf{i} - t^{1/2}\mathbf{j} + \mathbf{k}$ and $\mathbf{r}(0) = -\mathbf{i} + \mathbf{k}$

8. The position vector of a particle at time t is given by $\mathbf{r}(t) = 3t\ \mathbf{i} + 4\sin t\ \mathbf{j} + 4\cos t\ \mathbf{k}$

   (a) Determine the velocity vector at time $t = \pi$.

   (b) Determine the acceleration vector at time $t = \pi$.

   (c) Determine the speed at time $t = \pi$.

   (d) Determine the unit tangent vector $\mathbf{T}$ at time $t = \pi$.

   (e) Determine the principal unit normal vector $\mathbf{N}$ at $t = \pi$.

   (f) Determine the tangential component of acceleration at $t = \pi$.

   (g) Determine the normal component of acceleration at $t = \pi$.

   (h) Determine the curvature of the path at time $t = \pi$.

9. Represent the intersection of the two surfaces as a vector-valued function. You need not sketch the curve. Surface 1: $x - z^2 - y^2 = 0$ and surface 2: $z = 2y^2$

10. Find a set of parametric equations for the line tangent to the space curve generated by

$$\mathbf{r}(t) = <\ e^{-t},\ \sin t,\ 4t\ >$$

    at $t = 0$.

11. Find the open interval(s) on which the curve given by $\mathbf{r}(t)$ is smooth.

$$\mathbf{r}(t) = t^2\ \mathbf{i} + (e^t - t)\ \mathbf{j} + \mathbf{k}$$

12. Evaluate the definite integral $\int_0^{\pi/2} [2\cos t\ \mathbf{i} + 2\sin t\ \mathbf{j} + \mathbf{k}]\ dt$

13. Represent the plane curve $y = x^2 + 2$ by a vector valued function $\mathbf{r}(t)$.

14. Sketch the graph of the vector valued function $\mathbf{r}(t) = \sin(t)\mathbf{i} - 2\cos(t)\mathbf{j}$ for $0 \le t \le \frac{3\pi}{2}\pi$.

15. A baseball is hit 3 feet above the ground at 128 feet per second and at an angle of $\frac{\pi}{6}$ with respect to the ground. Assume that the only force acting on the ball after it is hit is that due to gravity. ($g = 32$ feet/(sec)^2).

   (a) Use the fact that acceleration is constant and given by $\mathbf{a}(t) = -g\ \mathbf{j}$ to derive a function for the position of the ball $\mathbf{r}(t)$ for any time t. Show your derivation below and write the result in the box.

   (b) What is the maximum height the ball reaches?

16. The DNA molecule has the shape of a double helix. The radius of each helix is about 10 angstroms (1 angstrom $= 10^{-8}$ cm). Each helix rises about 34 angstroms during each complete turn and there are about 2.9×10^8 complete turns, so the vector valued function defining each helix is

$$\mathbf{r}(t) = (10\cos t)\mathbf{i} + (10\sin t)\mathbf{j} + \left(\frac{34}{2\pi}t\right)\mathbf{k}, \quad 0 \le t \le 2\pi \times 10^8.$$

    Determine the length of the helix.