Sample Test 3

The actual exam will not be this long and WILL include some questions not represented here.

1. Assume f is a differentiable function of x and y.

   (a) **T** **F** If $\nabla f \neq 0$, then ∇f is orthogonal to the level curves of f.

   (b) **T** **F** If $\mathbf{u} =< 0, 1 >$ then $D_{\mathbf{u}} f(x, y) = f_x(x, y)$.

   (c) **T** **F** If $\mathbf{u} =< 1, 0 >$ then $D_{\mathbf{u}} f(x, y) = \lim_{\Delta x \to 0} \dfrac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$.

   (d) **T** **F** The direction of steepest **descent** of f is given by ∇f.

   (e) **T** **F** If $\nabla f \neq 0$, then ∇f is parallel to the level curves of f.

   (f) **T** **F** If $\mathbf{u} =< 0, 1 >$ then $D_{\mathbf{u}} f(x, y) = f_y(x, y)$.

   (g) **T** **F** $D_{\mathbf{u}} f = \nabla f \times \mathbf{u}$

   (h) **T** **F** If $\nabla f(x_o, y_o) = \overrightarrow{0}$, then (x_o, y_o) is a critical point.

   (i) **T** **F** The direction of maximum increase of f is given by ∇f.

2. If $\mathbf{v} =< 3, 4 >$ and $f(x, y) = x^2 + y^2$, then the directional derivative of f in the direction of $\mathbf{v}$ at the point (1,1) is

   (a) 70 (b) $\dfrac{14}{5}$ (c) $2\mathbf{i} + 2\mathbf{j}$ (d) $10\sqrt{2}$ (e) 5

3. The level curves of the function $f(x, y) = x^2 - y$ are

   (a) Circles (b) Parabolas (c) Ellipses (d) Hyperbolas (e) Lines

4. $$\lim_{(x,y)\to(0,0)} \frac{2\sin(x^2 + y^2)}{x^2 + y^2} =$$

   (a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$ (e) Does not Exist

5. An equation for the tangent plane to the surface $z = x^2 - y^2$ at the point (5,4,9) is:

   (a) $3x + 2y + z = 14$ (b) $x + 2y + 3z = 10$ (c) $10x - 8y - z = 9$

   (d) $x + 3y + 9z = 18$ (e) $x + 3y = 4$

6. If $f(x, y) = x^2y^2 + ax^2y + bxy^2 + cxy + d$ then $f_{xy} =$

   (a) $2xy^2 + 2axy + by^2 + cy$ (b) $4xy + 2ax + 2by + c$ (c) $2y^2 + 2ay$

   (d) $2x^2 + 2bx$ (e) $2x^2y + ax^2 + 2bxy + cx$

7. If $\mathbf{v} =< 3, -4 >$ and $f(x, y) = x^2 + xy + y^2$, then the directional derivative of f in the direction of $\mathbf{v}$ at the point (1,1) is

   (a) $\dfrac{-3}{5}$ (b) $\dfrac{\sqrt{2}}{2}$ (c) $\dfrac{3}{5}\mathbf{i}$ (d) $\dfrac{3}{5}\mathbf{j}$ (e) 3

8. Given $z = x^2 + y^3$ use the total differential to estimate the change in z as (x, y) goes from (1,1) to (1.1,1.2).

9. Given $w = xyz$, where $x = t^2$, $y = 2t$, and $z = e^{-t}$. Find dw/dt using the chain rule.

10. Suppose $f(x, y)$ is differentiable at all x and y. State the formal (limit) definition of the partial derivative of f with respect to y.

11. The radius r and height h of a cylinder are measured with possible errors of 2% and 4%. Approximate the maximum possible percent error in the calculated volume by using the total differential as an approximation to volume change. Note: Volume of a cylinder $= \pi r^2 h$.

12. The radius r and height h of a cylinder are increasing at a rate of 2 cm/min and 3 cm/min respectively. Find the rate of increase of the volume when $r = 5$ and $h = 10$. Use correct units in describing your answer. Give the answer exactly, NOT a decimal approximation.

13. Find a unit normal vector to the surface $x^2 + y^3 + 6z^3 = 5$ at the point (2,1,0).

14. Let $g(x, y) = 2x^2 + 3y^2 - 4x - 12y + 13$. Find the critical point(s) of g and determine whether there is a relative minimum, relative maximum, or saddle point at each.

15. Find the point on the plane $z = y - 2x - 3$ that is closest to the point (0,0,0). I suggest you minimize the square of the distance as opposed to the actual distance.

16. Find a unit normal vector to the surface $x^2 + y^2 + z^2 = 11$ at the point (3,1,1).

17. Let $g(x, y) = x^2 - 3xy - y^2$. Find the critical point(s) of g and determine whether there is a relative minimum, relative maximum, or saddle point at each.

18. A rectangular box **WITH NO TOP** is to be constructed which has a volume of 32 cubic feet. Find the dimensions of the box which minimize the surface area. Try this problem using Lagrange multipliers and without using Lagrange multipliers.

19. Use Lagrange multipliers to maximize $f(x, y) = e^{xy}$ subject to the constraint that $x^2 + y^2 = 8$.