Cheat Sheet for Final

- Method of variation of parameters for linear, second order, non-homogeneous equations. The particular solution is given by

$$\mathbf{Y}(t) = -y_1(t)\int \frac{y_2(t)\; g(t)}{W(y_1,y_2)(t)}\,dt + y_2(t)\int \frac{y_1(t)\; g(t)}{W(y_1,y_2)(t)}\,dt.$$

- Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	restrictions	
1	$\frac{1}{s}$	$s > 0$	(1)
e^{at}	$\frac{1}{s-a}$	$s > a$	(2)
t^n	$\frac{n!}{s^{n+1}}$	$s > 0$, n = positive integer	(3)
$\sin at$	$\frac{a}{s^2+a^2}$	$s > 0$	(4)
$\cos at$	$\frac{s}{s^2+a^2}$	$s > 0$	(5)
$\sinh at$	$\frac{a}{s^2-a^2}$	$s > \lvert a\rvert$	(6)
$\cosh at$	$\frac{s}{s^2-a^2}$	$s > \lvert a\rvert$	(7)
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$	$s > a$	(8)
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$	$s > a$	(9)
$u_c(t)$	$\frac{e^{-cs}}{s}$	$s > 0$	(10)
$u_c(t)f(t-c)$	$e^{-cs}F(s)$		(11)
$e^{ct}f(t)$	$F(s-c)$		(12)
$\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$		(13)
$\delta(t-c)$	e^{-cs}		(14)
$f'(t)$	$sF(s) - f(0)$		(15)
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$		(16)

- Fourier series representation of a periodic function with period $2\ L$.

$$f(x) = \frac{a_o}{2} + \sum_{m=1}^{\infty}\left(a_m\ \cos\frac{m\pi x}{L} + b_m\ \sin\frac{m\pi x}{L}\right)$$

where

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\ \cos\frac{n\pi x}{L}\,dx \quad \text{for } n = 0, 1, 2, \ldots,$$

and

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\ \sin\frac{n\pi x}{L}\,dx \quad \text{for } n = 1, 2, \ldots.$$

Beware that a_o is usually solved by replacing $\cos(\frac{0\pi x}{L})$ with 1.