

Group Problem for Math 251 (20 points towards Test 1)

Cover Page (staple this to your assignment)

Lake:

Group Members: There must be at least 2 and no more than 4 to a group.

- 1.
- 2.
- 3.
- 4.

Polluted Lake Problem

Consider a lake of constant volume V (in km^3) containing at time t an amount $Q(t)$ (in kg) of pollutant, evenly distributed throughout the lake with a concentration of $c(t)$, where $c(t) = Q(t)/V$. Assume that water containing a concentration k of pollutant enters the lake at a rate r , and that water leaves the lake at the same rate. At time $t = 0$, the concentration of pollutant is c_0 .

- Derive and write out the initial value problem for the **concentration** of pollutant in the lake.
- Solve this initial value problem for $c(t)$.
- What is the limiting concentration as $t \rightarrow \infty$?
- Let k , V , and r be given by the table below and the lake you are assigned. Use **Maple** to draw a direction field for the differential equation from part a. Does this direction field agree with your answer to part c?
- If the addition of pollutants to the lake is terminated ($k = 0$ for $t > 0$), determine the time interval T that must elapse before the concentration of pollutants is reduced to 50% of its original value; to 10% of its original value. Put your answer in the form of a sentence.
- Let $c_0 = 100$ and use **Maple** to graph $c(t)$ under the conditions of part (e). On this graph, indicate the values of T found in part (e) and verify that these correspond to the appropriate concentrations.

Lake	V ($\text{km}^3 \times 10^3$)	r (km^3/year)	k (kg/km^3)
Superior	12.2	65.2	10
Michigan	4.9	158	15
Erie	0.46	175	20
Ontario	1.6	209	25

Note on Format The two plots requested in parts (d) and (f) should be put onto **one** page. I don't want to see the Maple commands, just the plots properly labelled. **Presentation constitutes 5 of the possible 20 points.**