

Math 250, Section 1, TEST 1

Name:

Graphing calculators are allowed during this exam, **except TI-89's and TI-92's**. All non-trivial answers must be justified by relevant work. In these non-trivial cases, a correct answer without any work will receive **no credit**.

1. Solve the following initial value problems.

(30 pts)

(a) $y' - 2y = e^{2t}$, $y(0) = 2$.

your answer:

(b) $y' - y^2 \sin(x) = 0$, $y(0) = \frac{1}{2}$

your answer:

2. Solve the following differential equations. You need not solve y explicitly in terms of x . (25 pts)

(a) $\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^y}$

your answer:

(b) $(3x^2 - 2xy + 2) + (6y^2 - x^2 + 3)\frac{dy}{dx} = 0$

your answer:

3. A tank initially contains 100 liters of pure water. A mixture containing a concentration of γ g/liter of salt enters the tank at a rate of 2 liters/min, and the well-stirred mixture leaves the tank at the same rate. Answer the following questions. Keep in mind that the answers will depend on the unknown parameter γ . (25 pts)

(a) Let $Q(t)$ denote the grams of salt in the tank at any time t . Derive a differential equation which has $Q(t)$ as the solution. **Include an initial value.** your answer:

(b) Solve the initial value problem derived in part (a). your answer:

(c) What is the limiting value of Q as $t \rightarrow \infty$?

your answer:

4. Determine, **without solving the problem**, an interval in which the solution of the given differential equation is certain to exist. (10 pts)

$$(t - 3)y' + \sqrt{t}y = e^{2t}, \quad y(1) = 2$$

your answer:

5. Consider the differential equation $\frac{dy}{dt} = (y - 1)(3 - y)$. (10 pts)

(a) Make a rough sketch of the graph of y' versus y .

(b) Suppose $y(0) = 2$. **Without solving the differential equation**, determine what happens to y as t gets large. Explain your reasoning.

(c) Suppose $y(0) = 4$. **Without solving the differential equation**, determine what happens to y as t gets large. Explain your reasoning.