

Graphing calculators are allowed during this exam, **except TI-89's and TI-92's**. All non-trivial answers **must be justified by relevant work**. In these non-trivial cases, a correct answer without any work will receive **no credit**.

1. Consider the differential equation: $y'' + 5y' + 6y = 0$ (15 pts)

(a) Find the *general solution* of the differential equation.

your answer:

(b) Find the solution satisfying the initial conditions $y(0) = 0$ and $y'(0) = -3$.

your answer:

2. Consider the differential equation: $y'' - 4y' + 4y = 0$ (15 pts)

(a) Find the *general solution* to the differential equation.

your answer:

(b) Determine the possible behaviors of any non-zero solution as $t \rightarrow \infty$. Show work that justifies your answer.

your answer:

3. Consider the differential equation: $y'' + 2y' + 10y = 0$ (15 pts)

(a) Find the *general solution* to the differential equation.

your answer:

(b) Describe the behavior of any solution as $t \rightarrow \infty$. Show work that justifies your answer.

your answer:

4. If y_1 and y_2 are linearly independent solutions of $y'' + p(t)y' + q(t)y = 0$, prove that $c_1 y_1$ and $c_2 y_2$ are also linearly independent, provided that neither c_1 nor c_2 is zero. **Hint** Express the wronskian of the second set of solutions in terms of the wronskian of the first.

(10 pts)

5. Use the *method of undetermined coefficients* or *variation of parameters* to find a particular solution \mathbf{Y} of the nonhomogeneous equation; (15 pts)

$$y'' - 2y' - 3y = 6e^{2t}.$$

your answer:

6. Give an example of a non-homogeneous differential equation where the *method of undetermined coefficients* cannot be used and we would be forced to use the gruelling method of *variation of parameters*. (10 pts)

your answer:

7. Determine the **form** of the particular solution, **Y**, to the given differential equations. **Do not try to solve for the particular solution.** Note: This is the second step involved in *the method of undetermined coefficients*. Your answers should contain one or more undetermined coefficients. (20 pts)

(a) $y'' - y' - 2y = 2e^{-t}$

your answer:

(b) $y'' - y' - 2y = 3\sin(2t)$

your answer:

(c) $y'' - y' - 2y = 7t^2$

your answer:

(d) $y'' + 4y = \cos(2t)$

your answer: