

Math 250, Section 1, TEST 3

Name:

Graphing calculators are allowed during this exam, **except TI-89's and TI-92's**. All non-trivial answers **must be justified by relevant work**. In these non-trivial cases, a correct answer without any work will receive **no credit**.

1. Consider the nonhomogeneous differential equation: (20 pts)

$$y^{iv} - 2y'' + y = e^{-t} + e^{5t} + \cos t$$

where y^{iv} represents the fourth derivative of y .

- (a) Find the *general solution* of the corresponding *homogenous equation*.

your answer:

- (b) Find a suitable form for a *particular solution* $\mathbf{Y}(t)$ if the method of undetermined coefficients is to be used. Do not evaluate the constants.

your answer:

2. Find the radius of convergence (ρ) of the given power series.

(15 pts)

(a)
$$\sum_{n=0}^{\infty} \frac{n}{2^n} (x-1)^n$$

your answer:

(b)
$$\sum_{n=0}^{\infty} \frac{x^n}{n!}$$

your answer:

3. The differential equation $y'' - xy' - y = 0$ has two solutions defined by

(20 pts)

$$y_1(x) = 1 + \frac{x^2}{2} + \frac{x^4}{2 \cdot 4} + \frac{x^6}{2 \cdot 4 \cdot 6} + \dots = 1 + \sum_{n=1}^{\infty} \frac{x^{2n}}{2 \cdot 4 \cdot 6 \cdot \dots \cdot 2n}$$
$$y_2(x) = x + \frac{x^3}{3} + \frac{x^5}{3 \cdot 5} + \frac{x^7}{3 \cdot 5 \cdot 7} + \dots = x + \sum_{n=1}^{\infty} \frac{x^{2n+1}}{3 \cdot 5 \cdot 7 \cdot \dots \cdot (2n+1)}$$

(a) Verify that these two functions are linearly independent.

(b) Find the value of the constants a_0 and a_1 in the general solution

$$y = a_0 y_1 + a_1 y_2$$

which satisfy the initial conditions $y(0) = \frac{1}{2}$ and $y'(0) = 3$.

your answer:

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4. Solve the differential equation

(30 pts)

$$y'' + xy' + 2y = 0.$$

by means of a power series about the point $x_0 = 0$. Answer the following questions along the way and put these answers in the boxes provided.

(a) Determine the recurrence relation between a_{n+2} and a_n . your answer:

(b) Find the general solution to the differential equation.

your answer: