

Graphing calculators are allowed during this exam, **except TI-89's and TI-92's**. All non-trivial answers **must be justified by relevant work**. In these non-trivial cases, a correct answer without any work will receive **no credit**.

1. Find the singular point of the given differential equation and classify it as regular or irregular. Justify your classification. (10 pts)

$$x^2y'' + 2y' + 3y = 0$$

your answer:

2. Solve the following initial value problem. Assume $x > 0$.

(10 pts)

$$x^2y'' - 5xy' + 9y = 0 \quad y(1) = 3 \quad \text{and} \quad y'(1) = 2$$

your answer:

3. Find the inverse Laplace transform of the following functions.

(20 pts)

(a) $F(s) = \frac{2e^{-2s}}{s^2 - 4}$

your answer:

(b) $F(s) = \frac{s + 4}{s^2 - 2s + 5}$

your answer:

4. Find the Laplace transform of the following function.

(10 pts)

$$f(t) = \begin{cases} 0 & t < 2 \\ (t - 2)^3 & t \geq 2 \end{cases}$$

your answer:

5. Use the Laplace transform to solve the initial value problem

(20 pts)

$$y'' + 2y' - 3y = 0, \quad y(0) = 1, \quad y'(0) = 2$$

your answer:

6. Find the solution of the initial value problem

(30 pts)

$$y'' + 9y = u_2(t), \quad y(0) = 1, \quad y'(0) = 2$$

your answer: