The final exam will be composed of problems taken essentially from tests 1 - 5. An example such a sampling is given here.

1. For each differential equation, circle **all** solutions.

   - $y'' + 3y' + 2y = 0$

     $\phi_1(t) = te^{-t}$ $\quad \phi_2(t) = 3e^{-t}$ $\quad \phi_3(t) = 2e^{-t} + 4e^{-2t}$ $\quad \phi_4(t) = e^{-2t}$ $\quad \phi_5(t) = 3t^2$

   - $y'' + 2y' + 10y = 0$

     $\phi_1(t) = e^{-t}$ $\quad \phi_2(t) = e^{-t}\cos(3t)$ $\quad \phi_3(t) = \cos(3t)$ $\quad \phi_4(t) = 3\sin(t) + 2\cos(t)$ $\quad \phi_5(t) = 0$

   - $y'' - 4y' + 4y = 0$

     $\phi_1(t) = (3t+4)e^{2t}$ $\quad \phi_2(t) = e^{2t}$ $\quad \phi_3(t) = e^{-2t}$ $\quad \phi_4(t) = e^{2t}\cos(2t)$ $\quad \phi_5(t) = te^{2t}$

2. Determine the **form** of the particular solution, **Y**, to the given differential equation. **Do not try to solve for the particular solution.** Your answer should contain one or more undetermined coefficients.

   $y'' + 5y = \sin(\sqrt{5}\, t)$

3. Solve the following initial value problem.

   $y' - 2y = 4, \quad y(0) = 2.$

4. Solve the following differential equation. You need not solve y explicitly in terms of x.

   $\frac{dy}{dx} = \frac{-4x - 2y}{2x + 5y}$

5. Consider the initial value problem

   $$y'' - 2xy' - y = 0, \qquad y(0) = 1, \quad y'(0) = 2.$$

   Write out the first **four** nonzero terms in the series solution to this problem. You may use either the power series method or the Taylor series method to obtain these terms.

6. Use the Laplace Transform to solve the initial value problem.

   $y'' + 2y' - 3y = 0, \quad y(0) = 2, \quad y'(0) = 0$

7. Find the eigenvalues and eigenfunctions of the given boundary value problem. **You may assume** $\lambda \geq 0$. $y'' + \lambda y = 0, \quad y(0) = 0, \quad y'(2) = 0$

8. Consider the periodic function

$$f(x) = \begin{cases} x+3, & -3 \leq x < 0 \\ 3-x, & 0 \leq x < 3 \end{cases} \qquad f(x+6) = f(x).$$

Write out the first three nonzero terms in the Fourier series for this function.

9. Consider the following wave equation

$$4u_{xx} = u_{tt}$$

$$u(0,t) = 0, \quad u(10,t) = 0$$

$$u(x,0) = x(10-x), \quad u_t(x,0) = 0$$

Find the solution to this problem. If you express the solution in terms of an infinite sum, you need not evaluate the coefficients but must define them explicitly.

Cheat Sheet for Final

- Method of variation of parameters for linear, second order, non-homogeneous equations. The particular solution is given by

$$\mathbf{Y}(t) = -y_1(t)\int \frac{y_2(t)\,g(t)}{W(y_1,y_2)(t)}\,dt + y_2(t)\int \frac{y_1(t)\,g(t)}{W(y_1,y_2)(t)}\,dt.$$

- Laplace Transforms

$f(t) = \mathscr{L}^{-1}\{F(s)\}$	$F(s) = \mathscr{L}\{f(t)\}$	restrictions	
1	$\frac{1}{s}$	$s > 0$	(1)
e^{at}	$\frac{1}{s-a}$	$s > a$	(2)
t^n	$\frac{n!}{s^{n+1}}$	$s > 0$, n = positive integer	(3)
$\sin at$	$\frac{a}{s^2+a^2}$	$s > 0$	(4)
$\cos at$	$\frac{s}{s^2+a^2}$	$s > 0$	(5)
$\sinh at$	$\frac{a}{s^2-a^2}$	$s > \|a\|$	(6)
$\cosh at$	$\frac{s}{s^2-a^2}$	$s > \|a\|$	(7)
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$	$s > a$	(8)
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$	$s > a$	(9)
$u_c(t)$	$\frac{e^{-cs}}{s}$	$s > 0$	(10)
$u_c(t)f(t-c)$	$e^{-cs}F(s)$		(11)
$e^{ct}f(t)$	$F(s-c)$		(12)
$\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$		(13)
$\delta(t-c)$	e^{-cs}		(14)
$f'(t)$	$sF(s) - f(0)$		(15)
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$		(16)

- Fourier series representation of a periodic function with period $2\,L$.

$$f(x) = \frac{a_o}{2} + \sum_{m=1}^{\infty}\left(a_m\,\cos\frac{m\pi x}{L} + b_m\,\sin\frac{m\pi x}{L}\right)$$

where

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\,\cos\frac{n\pi x}{L}\,dx \quad \text{for } n = 0,1,2,\ldots,$$

and

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\,\sin\frac{n\pi x}{L}\,dx \quad \text{for } n = 1,2,\ldots.$$

Beware that a_o is usually solved by replacing $\cos(\frac{0\pi x}{L})$ with 1.