Math 251, Section 1, TEST 1 **ANSWERS**

1. Consider the differential equation $y' + p(t)y = g(t)$, the initial condition $y(t_o) = y_o$, and an open interval $I : \alpha < t < \beta$. (*10 pts)*

   (a) **T** **F** If p and g are continuous on I and t_o is in I, then there exists a unique solution to the differential equation which satisfies the initial condition.

   (b) **T** **F** If p and g are continuous for all t, then the **differential equation** has infinitely many solutions.

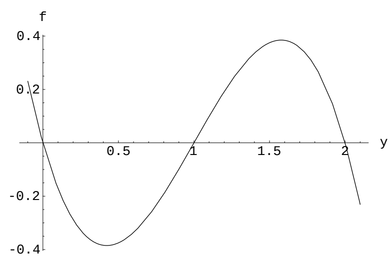
   (c) **T** **F** If $g(t) = 0$, and $y = \phi(t)$ is a solution to the **differential equation**, then $y = c\,\phi(t)$ is also a solution to the differential equation for any constant c.

   (d) **T** **F** If p is discontinuous at $t = \alpha$ then the solution must be discontinuous at $t = \alpha$.

   (e) **T** **F** If p and g are continuous for all t, it is possible that the solution may have a discontinuity at $t = \alpha$.

2. Consider the differential equation $y' = f(y)$ where f is plotted below. (*10 pts)*

   (a) Write two equilibrium solutions.



   (b) Classify their stability

3. Solve the following initial value problems. (*30 pts)*

   (a) $y' - 2y = t^2 e^{2t}, \quad y(0) = 2.$

   (b) $y' = \dfrac{1-2x}{y}, \quad y(1) = 2$

4. Solve the following differential equation. You need not solve y explicitly in terms of x. *(10 pts)*

$$\frac{dy}{dx} = \frac{-3x-2y}{2x+3y}$$

5. A tank initially contains 100 gallons of saline solution with a concentration γ pounds of salt per gallon . At time zero, fresh water is added to the tank at a rate of 3 gallons per minute and the mixed solution is allowed to leave the tank at the same rate. *(20 pts)*

   (a) Let $Q(t)$ denote the pounds of salt in the tank at any time t. Derive a differential equation which has $Q(t)$ as the solution. **Include an initial value.**

   (b) Solve the initial value problem derived in part (a).

   (c) What is the limiting value of Q as $t \to \infty$?

   (d) How long does it take for the the amount of salt in the tank to reach 25 percent of its original value?