Math 251, Section 1, TEST 2 **Answers**(*10 pts*)

1. Consider the differential equation (*10 pts)*

$$y'' + p(t)\,y' + q(t)\,y = 0,$$

the initial conditions

$$y(t_o) = y_o \quad \text{and} \quad y'(t_o) = y_o',$$

and an open interval

$$I : \alpha < t < \beta$$

which contains t_o, upon which p, and q are continuous.

(a) **T** **F** If y_1 and y_2 are both solutions of the differential equation then they form a fundamental set of solutions.

(b) **T** **F** There exists a unique solution to the differential equation which satisfies the initial conditions.

(c) **T** **F** If y_1 and y_2 are solutions to the differential equation, then $\phi(t) = \frac{y_1}{2} - 3y_2$ is also a solution.

(d) **T** **F** If y_1 and y_2 are zero at the same point in I, then they must form a fundamental set of solutions to the differential equation.

(e) **T** **F** If $y_o = y_o' = 0$, then the only solution to the initial value problem is $\phi(t) = 0$.

2. For each differential equation, circle **all** solutions. (*15 pts)*

- $y'' + 3y' + 2y = 0$

$\phi_1(t) = 3e^{-t}$ $\quad\phi_2(t) = 0$ $\quad\phi_3(t) = 3t^2$ $\quad\phi_4(t) = 2e^{-t} + 4e^{-2t}$ $\quad\phi_5(t) = te^{-t}$

- $y'' + 2y' + 10y = 0$

$\phi_1(t) = \cos(3t)$ $\quad\phi_2(t) = e^{-t}$ $\quad\phi_3(t) = 3\sin(t) + 2\cos(t)$ $\quad\phi_4(t) = e^{-t}\cos(3t)$ $\quad\phi_5(t) = 3t^2$

- $y'' - 4y' + 4y = 0$

$\phi_1(t) = e^{2t}$ $\quad\phi_2(t) = e^{-2t}$ $\quad\phi_3(t) = te^{2t}$ $\quad\phi_4(t) = (3t+4)e^{2t}$ $\quad\phi_5(t) = e^{2t}\cos(2t)$

3. Below is a differential equation and one solution; $y_1(t)$. Use the method of *reduction of order* to find a second solution. Hint: Let $y_2(t) = v(t)y_1(t)$ where $v(t)$ is an unknown function. *(15 pts)*

   $t^2y'' + 2ty' - 2y = 0, \quad t > 0, \quad y_1(t) = t$

4. Consider the differential equation $y'' + 2y' + 2y = 0$. *(15 pts)*

   - Find the *general solution* to the differential equation.
   - Find the solution satisfying the initial conditions $y(0) = 1$, and $y'(0) = -1$.
   - Describe the behavior of the solution as $t \to \infty$. Justify your answer.

5. Determine the **form** of the particular solution, **Y**, to the given differential equations. **Do not try to solve for the particular solution.** Note: This is the second step involved in *the method of undetermined coefficients*. Your answers should contain one or more undetermined coefficients. *(20 pts)*

   (a) $y'' + y' - 2y = 2e^{-2t}$

   (b) $y'' + y' - 2y = 3\cos(2t)$

   (c) $y'' + y' - 2y = 7t^2e^{3t}$

   (d) $y'' + 9y = \cos(3t)$

6. Use the *method of undetermined coefficients* or *variation of parameters* to find a particular solution **Y** of the given nonhomogeneous equation. $y'' + 2y' - 3y = 15e^{2t}$. *(15 pts)*