

1. Consider the nonhomogeneous differential equation: $y''' - y' = te^{-t} + 2\cos t$ where y''' represents the third derivative of y . (20 pts)

(a) Find the *general solution* of the corresponding *homogenous equation*.

(b) Find a suitable form for a *particular solution* $\mathbf{Y}(t)$ if the method of undetermined coefficients is to be used. Do not evaluate the constants.

2. Consider the initial value problem

$$y'' - 2xy' - y = 0, \quad y(0) = 2, \quad y'(0) = 0.$$

Obtain a series solution (write down the first three nonzero terms) by the following two different methods.

(a) The Power Series Method

(15 pts)

(b) The Taylor Series Method

(10 pts)

3. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{3^n}{n^2} x^n.$$

(5 pts)

your answer:

4. Consider the differential equation $x^2 y'' + xy' + \frac{y}{x} = 0$

(10 pts)

(a) What is the singular point of this differential equation.

your answer:

(b) Classify it as regular or irregular. (justify your answer)

your answer:

5. Find the general solution to the differential equation: $x^2 y'' + 3xy' + y = 0, \quad x > 0.$

(10 pts)

your answer: