

1. Find the Laplace transform of the following functions.

(20 pts)

$$(a) f(t) = \begin{cases} 0 & t < 2 \\ \cos 3(t-2) & t \geq 2 \end{cases}$$

$$(b) f(t) = e^{at} \cosh(bt)$$

2. Find the inverse Laplace transform of the following function.

(10 pts)

$$F(s) = \frac{s+2}{s^2+2s+5}$$

3. Use the Laplace transform to solve the initial value problem

(20 pts)

$$y'' + 2y' - 3y = 0, \quad y(0) = 2, \quad y'(0) = 1$$

4. Find the solution of the initial value problem

(20 pts)

$$y'' - 9y = \delta(t - 2), \quad y(0) = 0, \quad y'(0) = 0$$

5. Find the inverse Laplace transform; $f(t)$, of the function below using the **convolution integral**. Follow the steps.

$$F(s) = \frac{1}{s^2(s+2)}$$

(a) Express $f(t)$ as a convolution integral of two functions.

(10 pts)

(b) Carry out the integration.

(10 pts)