Math 251, Section 1, TEST 5 **Answers**

1. Find the eigenvalues and eigenfunctions of the given boundary value problem. **You may assume** $\lambda \geq 0$. (*20 pts)*

$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y'(\pi) = 0$$

2. Consider the periodic function (*20 pts)*

$$f(x) = \begin{cases} 0, & -1 \leq x < 0 \\ 1, & 0 \leq x < 1 \end{cases} \qquad f(x+2) = f(x).$$

(a) Write out the first three nonzero terms in the Fourier series for this function.

(b) Sketch the graph of the function to which the entire Fourier series would converge. Plot this on the interval $-1 \leq x \leq 1$.

3. Find the solution of the heat conduction problem. The solution is a finite Fourier series, write out all of the terms explicitly. (*20 pts)*

$$3u_{xx} = u_t, \quad 0 < x < 1, \quad t > 0;$$
$$u(0,t) = 0, \quad u(1,t) = 0, \quad t > 0;$$
$$u(x,0) = 5\sin(2\pi x) - 2\sin(3\pi x)$$

4. The method of separation of variables can be used to replace the following differential equation with a pair of ordinary differential equations. Find this pair of equations. (*10 pts)*

$$xu_{xx} - u_t = 0$$

5. Consider the following heat conduction problem with non-homogenous boundary conditions. (*10 pts)*

$$100u_{xx} = u_t, \quad 0 < x < 2, \quad t > 0;$$
$$u(0,t) = 10, \quad u(2,t) = 0, \quad t > 0;$$
$$u(x,0) = x^3 + 5x, \quad 0 < x < 2$$

(a) Find the steady-state solution.

(b) Find the boundary value problem that determines the transient solution. This is a differential equation, boundary conditions, and an initial condition.