Homework #1 Due: Wednesday September 17

MTHBD/CMPBD 423

Notes: Late submission: 25% reduction in grade. After Friday 9/19/2003 class-time: no credit. Hand in the exact number of pages requested for each problem (total 6 pages). Label problems, NAME, Stapled!!!!

1. *Taylor Polynomials and Error Analysis*:(one page) *(16 pts)*

   (a) Write the Taylor Series for e^{-x} about $x = 0$. This is the Maclaurin Series for e^{-x}.

   (b) Analytically determine a bound on the error from approximating $y = e^{-x}$ with it's linear approximation over the interval [0,1]. You should assume that $e^{-\eta} <= 1$ over this interval.

   (c) Determine the minimum degree Maclaurin polynomial for e^{-x} required to ensure that the error is less than 10^{-6} over the interval [0,1].

   (d) Prove that the error in approximating $y = e^{-x}$ with its linear approximation is $\mathcal{O}(x^2)$ (big O of x^2) as $x \to 0$.

2. *Plotting Taylor Polynomials and Errors*:(one page) *(10 pts)*

   You may wish to download **PlotExpTaylorError.m** from the class website and edit it for this problem making sure the graphs are labelled correctly. **All graphs should have your name in the title**. Using the subplot command in MATLAB generate the following two graphs on one figure.

   upper graph: Plot $\sin(x)$, and the Taylor (Maclaurin) polynomials: $P_1(x)$, $P_3(x)$, and $P_5(x)$ over the interval [0,2].

   lower graph Plot the associated errors for each polynomial over the same interval.

   On the same piece of paper briefly describe the relationship between:

   (a) The error and the distance of x from zero.

   (b) The error and the degree of the approximating polynomial.

3. *Nested Multiplication for Polynomial Evaluation*: (one page) *(10 pts)*

   Using the program **PolyEval.m** from the web-site and the function file **nested.m** that you have written, find a polynomial and a value of x_1 such that the results of evaluating the polynomial at x_1 with the traditional method differs from that utilizing nested multiplication. Hand in a print out of the command window showing only the run of PolyEval.m that results in a nonzero difference in evaluation techniques.

4. *Improving the Quadratic Formula*: (3 pages) *(14 pts)*

   In class, we discussed the potential problems associated with subtracting two nearly equal numbers and how to resolve this in the case of the quadratic formula when $b > 0$ and $b \approx \sqrt{b^2 - 4ac}$. Determine a way to resolve this same problem if $b < 0$. You may assume that $b^2 - 4ac > 0$.

   Write two MATLAB program functions; **quad1(a,b,c)** and **quad2(a,b,c)**. Which find the solutions to the equation

   $$ax^2 + bx + c = 0.$$

   The first function should compute the roots with the traditional quadratic formula. The second function should avoid subtracting approximately equal numbers based on the sign of b. Use both functions to solve for the coefficients given on the next page.

   Hand in paper copies of quad1(a,b,c) and quad2(a,b,c), and complete the next page.

NAME:

First set of coefficients: $a = 10^{-6}, \quad b = 10, \quad c = 10^{-6}$

	quad1(a,b,c)	quad2(a,b,c)
root 1		
root 2		

Second set of coefficients: $a = 10^{-6}, \quad b = -10, \quad c = 10^{-6}$

	quad1(a,b,c)	quad2(a,b,c)
root 1		
root 2		

Explain why there is a difference in the two (analytically) equivalent methods.