

Homework #2 Due: Friday October 3 MTHBD/CMPBD 423

Notes: Late submissions: 25% reduction in grade if the assignment is submitted after the due date but before the following class meeting time. No credit after that. Hand in the exact number of pages requested for each problem. Label problems, NAME, Stapled!!!!

1. *Bisection Method*:(one page) (10 pts)

Consider the bisection method which brackets a root r of $f(x) = 0$ with an initial interval of $[a_0, b_0]$, and $c_n = (a_n + b_n)/2$. Given $\epsilon > 0$, find N such that $|c_n - r| < \epsilon$ for all $n > N$.

2. *Analysis of Newton's method and fixed point iteration*:(one page) (10 pts)

Recall that for a simple zero r of f : $f(r) = 0$ and $f'(r) \neq 0$, and an initial x_0 chosen *close enough* to r , the sequence generated by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = g(x_n)$$

generates a sequence of x_n 's satisfying

$$|e_{n+1}| \rightarrow K_1 |e_n|^2 \quad \text{where} \quad K_1 = \left| \frac{f''(r)}{2f'(r)} \right|,$$

where $e_n = x_n - r$.

Similarly, we showed that

$$|e_{n+1}| \rightarrow K_2 |e_n|^2 \quad \text{where} \quad K_2 = \left| \frac{g''(r)}{2} \right|.$$

Show that $K_1 = K_2$ provided $f'''(x)$ exists on some interval about r .

3. *Newton's method sensitivity to initial guess*: (2 pages) (10 pts)

Write a MATLAB function file called **newton(fun,dfun,xo,tol)** which attempts to find the root of **fun** with an initial guess of **xo**. It stops when either $|x_n - x_{n-1}| < \mathbf{tol}$, or $|f(x_n)| < \mathbf{tol}$. Be sure to include a maximum number of iterations and print a warning if the number of iterations reaches this number because the result could very well be garbage.

Write a program file that creates a vector of x-values starting at 0 and ending at 4 spaced .02 units apart: $\vec{x} = [0, 0.02, 0.04, \dots, 3.98, 4.0]$. These will be used as initial guesses into the function you wrote where **fun** = $f(x) = x^3 - 5.56x^2 + 9.1389x - 4$, **dfun** = $f'(x)$, and **tol** = 10^{-8} .

Generate two graphs in the same figure using the **subplot** command:

- (a) Top graph: $f(x)$ versus x
- (b) lower graph: The root found by **newton(f,f',x,tol)** versus x .

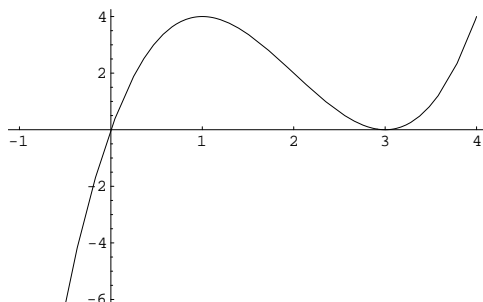
On the same page explain why Newton's method does not always converge to the root closest to the initial guess. Give an example based on the two graphs displayed on this page. Also hand in a paper copy of your **newton(fun,dfun,xo,tol)** function.

4. Orders of convergence: (2 pages)

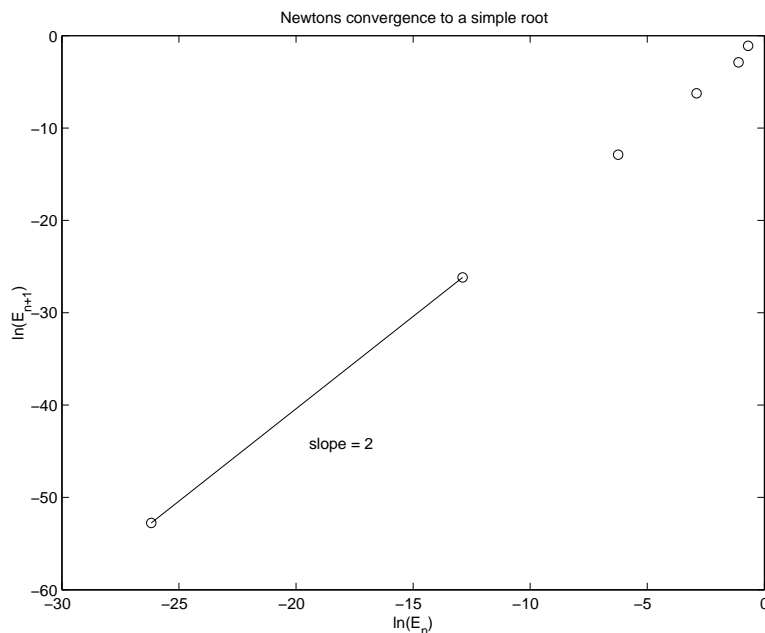
(20 pts)

Consider the function

$$f(x) = x(x - 3)^2.$$



It was shown in class that Newton's method converges quadratically to a simple root. This means if $f(r) = 0$ and $(f'(r) \neq 0)$ then Newton's method will produce a sequence x_n that converges to r in such a way that if $E_n = |x_n - r|$, then $E_{n+1} \leq CE_n^2$ provided x_n is *close enough* to r . Replacing the \leq with $=$ and taking the natural logarithm of both sides of the equation yields $\ln(E_{n+1}) = \ln(C) + 2 \ln(E_n)$. This means that the relationship between $\ln(E_{n+1})$ and $\ln(E_n)$ is linear with slope 2. The plot below displays this relationship for the Newton iterates converging to $r = 0$ starting with $x_1 = 0.5$ in the function above.



For this problem I want you to print up similar graphs verifying any two of the following three statements.

- (a) Newton's method converges linearly to $r = 3$ for $x_1 = 2$.
- (b) The secant method converges to $r = 0$ for $x_1 = 0.5$ and $x_2 = 0.25$ with order of convergence approximately 1.62.
- (c) Newton's method adapted for a double root converges quadratically to $r = 3$ for $x_1 = 2$.

Hand in each plot on a separate sheet of paper.