Homework #4 Due: Friday November 21

MTHBD/CMPBD 423

Be sure to read the Homework Rules. If you need a copy let me know. I will take off points according to those rules. You may discuss these problems with your classmates (only) but you cannot share code or copy proofs. If you do, I will consider it a violation of academic integrity.

1. Interpolating data with three forms of polynomials. *(10 pts)*

   You are asked to find the coefficients of the 4th degree polynomial which interpolates the following data

x	0	2	4	6	8
y	15	-17	-125	-285	-377

   - Vandermonde Method: Find the coefficients $(a_0, a_1, \ldots, a_n)$ of the interpolating polynomial in the form:
   $$p_n(x) = a_o + a_1 x + a_2 x^2 + \ldots + a_n x^n$$
   Find these coefficients using the Vandermonde matrix.
   - Lagrange interpolation polynomial: Find the coefficients $(b_0, b_1, \ldots, b_n)$ of the interpolating polynomial in the form:
   $$p_n(x) = b_0\, \ell_0(x) + b_1\, \ell_1(x) + \ldots + b_n\, \ell_n(x) \quad \text{where} \quad \ell_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$
   - Newton interpolation polynomial: Find the coefficients $(c_0, c_1, \ldots, c_n)$ of the interpolating polynomial in the form:
   $$p_n(x) = c_0 + c_1(x - x_0) + c_2(x - x_0)(x - x_1) + \ldots + c_n(x - x_0)(x - x_1)\cdots(x - x_{n-1})$$
   Use the method of divided differences to calculate these coefficients.

   For this problem you are to hand in the completed table on the next page. There is an example on page 311 that may be used to aid in checking your algorithms.

2. Do numbers 27 in Section 6.1 of the text. This should take one page. *(10 pts)*

3. Approximate $f(x) = \frac{1}{1+x^2}$ with $P_{10}(x)$ over the interval $[-5, 5]$ using 11 equally spaced nodes where $x_0 = -5$. Plot $f(x)$ and $P_{10}(x)$ over the vector $x = -5 : .1 : 5$. Now superimpose on this a graph of $P_{10}(x)$ using the Chebyshev nodes plotted over the same x values. So the figure should contain 3 graphs: The function, The polynomial interpolating 11 equally spaced nodes, and the polynomial interpolating 11 Chebyshev nodes. Clearly label each curve. You may use MATLAB's **polyfit** and **polyval** functions if you want but you have to learn how to use them properly. *(10 pts)*

4. Do number 26 in Section 6.2 of the text. **Hint** Interpolate f at $x_0 = x$, $x_1 = x + h$, and $x_2 = x + 2h$ with a quadratic Newton Interpolating Polynomial. You do not need any code. This should take one page. *(10 pts)*

5. **Cubic Spline Interpolation, periodic in 2 dimensions** *(10 pts)*

(a) Go to the course website and download **annulusData.m**. This gives the x and y coordinates of 7 piezo-electric crystals placed on the annulus of the mitral valve of a sheep. The third dimension is eliminated by taking the projection of the 3-D positions onto the plane of best fit. This has already been done. The seventh set of coordinates is actually a repeat of the first. So there are really only 6 crystals.

Consider the parametrization of the curve describing the mitral valve annulus

$$u(t) = (x(t), y(t))$$

where

$$\begin{array}{rcccl} u(0) & = & (x(0), y(0)) & = & (x_0, y_0) \\ u(1) & = & (x(1), y(1)) & = & (x_1, y_1) \\ \vdots & = & \vdots & = & \vdots \\ u(5) & = & (x(5), y(5)) & = & (x_5, y_5) \\ u(6) & = & (x(6), y(6)) & = & (x_6, y_6) = (x_0, y_0) \end{array}$$

(b) Perform cubic spline interpolation for periodic data on $x(t)$ and $y(t)$ individually. With this parametrization, let $h_i = t_{i+1} - t_i = 1$ for all i. You now have the cubic spline for $u(t)$. Choose a uniform time steps of $\Delta t = 0.1$ and plot $u(t) = (x(t), y(t))$ at these times as t goes from 0 to 6.

You may find it helpful to download and use the function file **findint.m**, available from the class web-site. It takes a vector of nodes (interval endpoints), and a value of x as input. It returns the interval number (starting at one) in which the number x lies. Due to the simplicity of the independent variable t this may not be needed. For example if $t = 0.3$, you would use $u(t) = u_0(t)$, if $t = 4.3$ you would use $u(t) = u_4(t)$.

(c) Now superimpose the plot of the spline on the original data. Make sure the original data points are clearly visible (use circles).

PAGE 1

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Table for Problem number 1

type	i = 0	i = 1	i = 2	i = 3	i = 4
a_i					
b_i					
c_i					

Hand In:

- **Page 1:** This page with the table completed.
- **Page 2:** Problem 27 from Section 6.1
- **Page 3:** The Graph from # 3 with all three curves clearly labelled.
- **Page 4:** Problem 26 from Section 6.2.
- **Page 5:** The annulus graph with cubic spline interpolating polynomial.
- **Pages** > 5**:** Code for problems 1,3, and 5 in that order.