Homework #5 Due: Thursday December 18

MTHBD/CMPBD 423

Be sure to read the Homework Rules. If you need a copy let me know. I will take off points according to those rules. You may discuss these problems with your classmates (only) but you cannot share code or copy proofs. If you do, I will consider it a violation of academic integrity.

1. **Fourier Coefficients (Transform) of a Continuous Function** *(10 pts)*

   Cardiac Output may be approximated by Stretching or Compressing the function

   $$Q(t) = \sin^n(\pi t)\ \cos(\pi t - \phi)$$

   where $n = 13$ and $\phi = \pi/10$. This function has period $P = 1$ and Q the flow is in ml/(unit time).

   Recall that a continuous function of period P has the fourier representation

   $$f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty} \left[A_n\ \cos(n\,\omega\,x)\ +\ B_n\ \sin(n\,\omega\,x)\right]. \tag{1}$$

   where

   $$A_n = \frac{2}{P}\int_0^P f(x)\cos(n\,\omega\,x)\,dx \tag{2}$$

   $$B_n = \frac{2}{P}\int_0^P f(x)\sin(n\,\omega\,x)\,dx \tag{3}$$

   and $\omega = \frac{2\pi}{P}$.

   Your assignment: Find the Fourier coefficients A_n and B_n for $0 \le n \le 8$ in the Fourier series representation of $Q(t)$.

   The integrals required to calculate these coefficients must be evaluated numerically. You may use any of the composite Newton-Cotes formulas (except the trapezoid rule) or Gaussian Quadrature. You may also interpolate points with a cubic spline and then integrate this. In any of these methods let $h \le 0.01$.

   Complete the table for problem number 1.

2. **Fourier Coefficients (Transform) of Discrete Data.** *(15 pts)*

   Go to the class website and download **FlowDataOne.m**. This is a 110 x 2 matrix. The first column is time in minutes and the second column is flow in ml/min in the ascending aorta of a sheep. The first and last flow value are the same. This data represents one period of a periodic function where the period is given by the last time element.

   Your assignment:

   - Find the Fourier coefficients A_n and B_n in the series representing this data for $0 \le n \le 8$. Since the time steps are not uniform the most practical method of numerical integration is the trapezoid rule, so use this. Fill in the table for this problem.
   - Plot the resulting finite Fourier series approximation from this data on top of the actual data itself. Use a curve for $F_N(t)$ and dots for the data. Keep increasing the number of harmonics (terms) until the curve appears to run through the data points. Print this one with the number of harmonics in the title.

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3. Derive a general formula for the optimal step size (h_{opt}) using the $\mathbf{O}(h^4)$ central differencing approximation to the derivative.

(10 pts)

4. Demonstrate that the above formula is valid by plotting the $\log_{10}$(h) -vs- log_{10}(error) for the first derivative approximations of $f(x) = e^x$ evaluated at $x_i = 1$. Set ϵ (the maximum round-off error) to be 10^{-15} and consider the differentiation interval to be $[0, 2]$.

(10 pts)

5. What is the slope of this curve before round-off error starts adversely effecting the approximation and why is this appropriate? Put this information and discussion on the same page as the graph.

(5 pts)

MTHBD/CMPBD 423 Homework # 4: Name: ....

Table for problem # 1:

Numerical Integration Method was ....

index	A_k	B_k
k = 0		
k = 1		
k = 2		
k = 3		
k = 4		
k = 5		
k = 6		
k = 7		
k = 8		

Are you on the right track for # 1? $A_1 \approx -0.113279$, $B_3 \approx 0.0498051$ and $A_6 \approx 0.000528105$.

Table for problem # 2:

index	A_k	B_k
k = 0		
k = 1		
k = 2		
k = 3		
k = 4		
k = 5		
k = 6		
k = 7		
k = 8		

Hand In:

- **Page 1:** This page with the tables completed.
- **Page 2:** The graph associated with problem number 2
- **Page 3:** The derivation of the optimal step size in problem 3.
- **Page 4:** The graph for problem 4 with the answer to problem 5 below it.
- **Pages** > 4**:** Code for problems 1,2, and 4 in that order.