Homework #2 Due: Friday October 7 MTHBD/CMPBD 423

1. Create a folder in your folder in my p-drive called YourlastnameHW2. All programs should be dropped in that folder. I do not want paper copies. I will only run the first program (bisection method) but the others (numbers 4 and 5) must be dropped so that I can verify you did it.

2. MATLAB Program for the bisection method: *Yourlastname*B.m (*5 pts*)

   This program will use the bisection method to approximate where the graphs of $y = 3x$ and $y = e^x$ intersect. This program should prompt the user for a_0, b_0, and Tol. The first two determine the initial interval. (Tol) will determine the maximum tolerated error in the final approximation. The program should first return (print to the command window) your name. Secondly it should return $n =$ the number of iterations necessary for the approximation to satisfy this condition. The program should thirdly return the approximation $= c_n$. The approximations should be generated by $c_k = (a_k + b_k)/2$. You should put in a safety stopping criterion so that the program will not run more than 40 iterations. If it takes more than 40 iterations the program should print a message to the screen and quit (I will check this).

3. *Analysis of Newton's method and fixed point iteration:*(one page) (*5 pts*)

   Recall that for a simple zero r of f: $f(r) = 0$ and $f'(r) \neq 0$, and an initial x_0 chosen *close enough* to r, the sequence generated by

   $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = g(x_n)$$

   generates a sequence of x_n's satisfying

   $$|e_{n+1}| \to K_1 |e_n|^2 \quad \text{where} \quad K_1 = \left| \frac{f''(r)}{2f'(r)} \right|,$$

   where $e_n = x_n - r$.

   Similarly, we showed that

   $$|e_{n+1}| \to K_2 |e_n|^2 \quad \text{where} \quad K_2 = \left| \frac{g''(r)}{2} \right|.$$

   Show that $K_1 = K_2$ provided $f'''(x)$ exists on some interval about r.

4. *Newton's method sensitivity to initial guess:* (one page) (*5 pts*)

   Write a MATLAB *function* file called **newton.m** with the first line

   function r = newton(x0)

   which returns r, an approximation to a zero of $f(x) = x^3 - 5.56x^2 + 9.1389x - 4$ when given an initial guess of **x0**. It stops when either $|x_n - x_{n-1}| < 10^{-8}$, or $|f(x_n)| < 10^{-8}$.

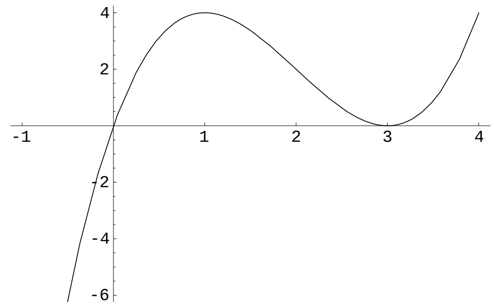
   In another program, create a vector of x-values starting at 0 and ending at 4 spaced .02 units apart: $\vec{x} = [0,\ 0.02,\ 0.04,\ \ldots\ ,\ 3.98,\ 4.0]$. These will be used as initial guesses. Generate two graphs in the same figure using the **subplot** command. The top graph should illustrate $f(x)$ versus x. The lower graph should illustrate the root found by **newton(x)** versus x.

   Print this figure and circle an x value on the lower graph where Newton's method did not converge to the nearest zero and briefly describe why this happened. Drop **newton.m** in my p-drive.

5. *Orders of convergence*: (1 page) *(5 pts)*

Consider the function

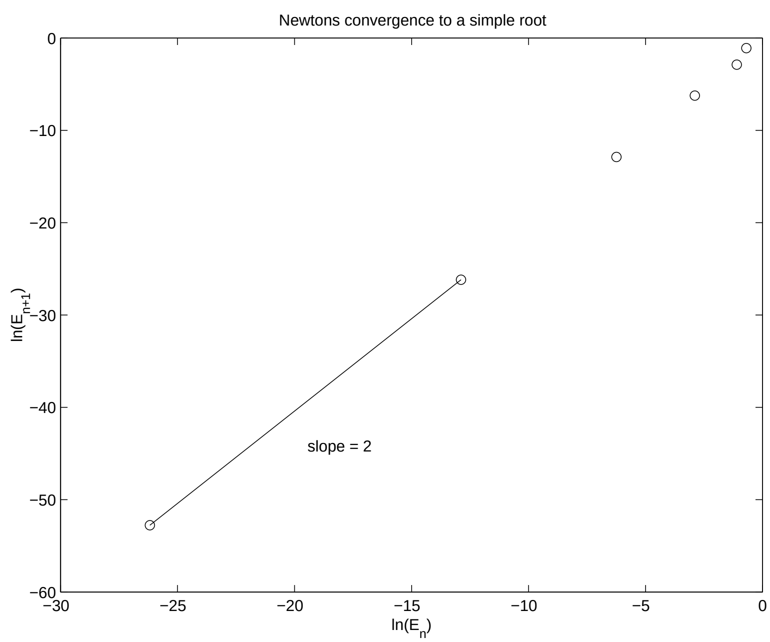
$$f(x) = x(x-3)^2.$$



It was shown in class that Newton's method converges quadratically to a simple root. This means if $f(r) = 0$ and $(f'(r) \neq 0)$ then Newton's method will produce a sequence x_n that converges to r in such a way that if $E_n = |x_n - r|$, then as n gets large,

$$E_{n+1} \to K E_n^2$$

where K is defined in problem 2. Taking the natural logarithm of both sides of the equation yields $ln(E_{n+1}) \to ln(K) + 2\, ln(E_n)$. This means that the relationship between $ln(E_{n+1})$ and $ln(E_n)$ is linear with slope 2 as n gets large. The plot below displays this relationship for the Newton iterates converging to $r = 0$ starting with $x_0 = 0.5$.



For this problem I want you to print up a similar graph verifying one of the following statements. You choose, but not both please.

(a) Newton's method converges linearly to $r = 3$ for $x_0 = 2$.

(b) The secant method converges to $r = 0$ for $x_0 = 0.5$ and $x_1 = 0.25$ with order of convergence approximately 1.62.

Hand in a paper print of the plot being sure to label it accordingly. Drop the generating MATLAB file(s) in the HW2 folder in my P-Drive.