

1. Interpolating data with three forms of polynomials. (10 pts)

You are asked to find the coefficients of the 4th degree polynomial which interpolates the following data

x	0	2	4	6	8
y	15	-17	-125	-285	-377

- Vandermonde Method: Find the coefficients (a_0, a_1, \dots, a_n) of the interpolating polynomial in the form:

$$p_n(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n$$

Find these coefficients using the Vandermonde matrix.

- Lagrange interpolation polynomial: Find the coefficients (b_0, b_1, \dots, b_n) of the interpolating polynomial in the form:

$$p_n(x) = b_0 \ell_0(x) + b_1 \ell_1(x) + \dots + b_n \ell_n(x) \quad \text{where} \quad \ell_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^n \frac{x - x_j}{x_i - x_j}$$

2. For the interpolation nodes(x-values) -1, 1, 3, 4 plot the Lagrange basis functions $l_i(x)$ for $0 \leq i \leq 3$ on the same axes. Describe the key features of these curves that are critical for their use in interpolating data points.