Homework #5 Due: Friday November 4 MTHBD/CMPBD 423

1. Use the error for the composite trapezoid rule to determine the minimum number of intervals needed to approximate
$$\int_0^5 \cos^2(x)\, dx$$
with an error less than 10^{-4} using the trapezoid rule. This is not a 'trial and error' question. You can determine this number analytically though you may need a calculator at the end.

(6 pts)

2. (number 19 from 5.2) Show that there exist coefficients ω_0, ω_1, ..., ω_n depending on x_0, x_1, ... , x_n and on a, b such that
$$\int_a^b p(x)\, dx = \sum_{i=0}^{n} \omega_i\, p(x_i)$$
for all polynomials p of degree $\leq n$. Hint: Use the Lagrange form of the interpolating polynomials.

(6 pts)

3. I made a modestly convincing argument that the error for the closed form trapezoid rule had the form:

(8 pts)

$$E(h) = \int_x^{x+h} f(x)\, dx - \frac{h}{2}(f(x) + f(x+h)) = -\frac{f''(\eta)h^3}{12} \quad \text{where } \eta \in (x, x+h)$$

Demonstrate that this is true by doing a $log|E|$ -vs- $log|h|$ plot for
$$\int_1^{1+h} \cos x\, dx.$$

Create a vector of 16 h values where $h_i = 2^{-i}$ for $i = 0, 1, \ldots, 15$. For each h_i, you know the exact solution to the above integral, approximate it using the trapezoid rule for one step, and let E_i be the error. Now plot $log(E_i)$ versus $log(h_i)$. Use the common log (base 10). The relationship should be linear with slope 3.

Hand in a paper copy of the graph you generated, be sure it is well labelled and state the slope relating the last two points on the line (last being the smallest two h's).