Homework #1 Due: Friday January 27

(50 pts)

MTHBD/CMPBD 424

1. Consider the system of linear equations defined by $Ax = b$ where

$$A = \begin{pmatrix} -4 & 1 & 1 & 1 \\ 1 & -4 & 1 & 1 \\ 1 & 1 & -4 & 1 \\ 1 & 1 & 1 & -4 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}.$$

   We are going to solve this system using three iterative methods: the Jacobi method, the Gauss-Seidel method, and the Gauss-Seidel with over-relaxation. We will compare the various rates of convergence under various changes in problem parameters.

   In all cases, the initial guess to the solution will be $x_o = [0,0,0,0]^{Tr}$ and the 2-norm should be used in measuring the residual.

   For the Jacobi method you may copy the function file I gave in class named **JFF.m**. You should write 2 other function files **GSFF.m** (Gauss-Seidel function file) and **GSORFF.m**(Gauss-Seidel Over Relaxation function file). The first line in each will be

   function [its, sol] = GSFF(A,b,xo,tol,normtype)

   and

   function [its, sol] = GSORFF(w,A,b,xo,to,normtype)

   Notice the GSORFF function file contains an extra input: w. This is the specific relaxation factor. All of these functions may be called from the same program file similar to **JPF.m** which I passed out in class. Drop these two files in your folder in my P-drive.

   (a) Fill out the table on the next page. In each square put in the number of iterations required to obtain a residual with norm less than the given tolerance. Generalize these results in a short paragraph.

   (b) As your table should display the over-relaxation method (with relaxation factor $\omega = 1.3$) requires fewer iterations than either of the previous two for a given tolerance. This however is not the optimal ω for this problem. Find ω_{opt} or a range of such optimal values for this problem. Provide a table or graph verifying your conclusion.

   (c) Increase the size of the system of equations to a 10 x 10 system. This means A is a 10 by 10 matrix with -10's on the diagonal and ones everywhere else, b is a vector of 10 ones, and let x_0 equal the appropriate sized vector of all zeros. Find ω_{opt} or a range of such optimal values for this problem and provide a table or graph verifying your conclusion. It should be greater than that found in the previous problem.

2. In class we showed that the Jacobi iteration method is equivalent to the more general form

$$Qx^{(k+1)} = (Q - A)x^{(k)} + b$$

   where Q is the diagonal matrix with elements taken from the diagonal of A. We also showed that such an algorithm produces a sequence which converges to the solution of $Ax = b$ provided $||I - Q^{-1}A|| < 1$ for some norm. Your assignment is to prove that $||I - Q^{-1}A||_\infty < 1$ when Q is that for the Jacobi method and A is diagonally dominant.

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Table for Problem number 1: For each method and tolerance fill in the number of iterations required for the norm of the residual to be less than the tolerance given.

	$tol = 10^{-6}$	$tol = 10^{-10}$	$tol = 10^{-12}$
Jacobi			
Gauss-Seidel			
Gauss-Seidel (over-relaxed where $\omega = 1.3$)			

Generalize how the Gauss-Seidel and over-relaxed Gauss-Seidel methods are better than the Jaocobi method based on this data.

Hand In:

- **Page 1:** This page with the table completed and generalizations made.
- **Page 2:** The answer to problem 1(b) with a verifying table or graph.
- **Page 3:** The answer to problem 1(c) with a verifying table or graph
- **Next page(s):** The answer to problem 2.
- **Remember:** Drop GSFF.m and GSORFF.m in your folder in my P-drive.