Homework #3 Due: Friday February 17

(50 pts)

MTHBD/CMPBD 424

1. Suppose that a differential equation is solved numerically on an interval $[a, b]$ and at each step you get a local truncation error, e_i satisfying $|e_i| = k_i h^p$ where h is the step size $= \frac{b-a}{N}$ and $k_i > 0$. Assuming that the only error is this local truncation error at each step, (ie no propagated error) show that there is $K > 0$ such that the global truncation error, E, satisfies $|E| \leq (b-a)Kh^{p-1}$. That is, give a weak proof as to why a local truncation error of order p results in a global error of order $p - 1$.

   We showed this for the case of $p = 2$ in class. We also did a much more complicated proof to show that this method had a global discretization error $= \mathcal{O}(h)$. I am asking for the simpler case only with p arbitrary.

2. Consider the initial value problem

$$x' = x + t \qquad x(0) = 0$$

   We numerically solved this problem in class using Euler's method and the Taylor series method of order 2. I want you to solve this problem using Euler's method and the Taylor series method of order 3. The exact solution is $x = e^t - (t + 1)$. I want you to find the numerical solution for $0 \leq t \leq 2$.

   Hand in

   (a) A page describing the steps in the Taylor series method of order 3 for this particular problem.

   (b) A graph of the exact solution, Euler's method and the Taylor series method of order 3 all on the same graph for $0 \leq t \leq 2$. For these let $h = .1$.

   (c) Create a vector of step sizes h=$[2^{-3}, 2^{-4}, \ldots, 2^{-10}]$. Let E_h represent the absolute value of the error at $t = 2$ from using Euler's method to approximate the solution with a step size of h. Hand in a graph of $\log_{10}(E_h)$ versus $\log_{10}(h)$ over the complete range of h values. Be sure to put $\log_{10}(h)$ on the x-axis and $\log_{10}(E_h)$ on the y-axis of your graph. On the same page comment on the slope of this curve and what this implies about the error of Euler's method.

   (d) Do the same as part (C) for the Taylor Series Method of Order 3.

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1. Put your proof here.

Hand In:

- **Page 1:** This page with the answer to problem 1.
- **Page 2:** The answer to problem 2(a).
- **Page 3:** The graph from problem 2(b).
- **Page 4:** The graph and comments from problem 2(c).
- **Page 5:** The graph and comments from problem 2(d).
- **Page $\geq$ 6:** The MATLAB files for number 2 should be placed in my P-drive.